Comment on “Magnus and Iordanskii Forces in Superfluids”

In a recent Letter, Wexler [1] concluded that there can be no transverse force on a quantized vortex proportional to the normal fluid velocity. We wish to point out that this conclusion conflicts with the experimental facts for vortices in superfluid ^{3}He and ^{4}He, irrespective of any theory offered to explain those facts, as in an earlier Comment [2].

Wexler notes that from Galilean invariance (which is certainly applicable to liquid helium, though not to superconductors) the most general transverse force per unit length on a vortex of circulation κ in the $\hat{\mathbf{z}}$ direction can be written as

$$\mathbf{F} = A\hat{\mathbf{z}} \times (\mathbf{v}_V - \mathbf{v}_s) + B\hat{\mathbf{z}} \times (\mathbf{v}_V - \mathbf{v}_n), \qquad (1)$$

where $\mathbf{v}_V$ is the vortex velocity. He then gives an elegant thermodynamic argument for the magnitude of the force proportional to $\mathbf{v}_s$ to show that $A = \rho_s \kappa$, and combines this with the result of Thouless, Ao, and Niu [3] (referred to as TAN) for the force proportional to $\mathbf{v}_V$ to conclude that $B \equiv 0$.

Measurements of the quantum of circulation [4] demonstrate the first term in Eq. (1) for a vortex trapped on a wire, but the effect of the normal fluid is a minor one in these experiments, and moreover is given by ordinary viscous drag on the wire in this case. The force exerted by the normal fluid on a free vortex is best measured in mutual friction experiments, where it is equal and opposite to the force exerted by the superfluid.

A nonzero value of Wexler’s B (not to be confused with the conventional dissipative mutual friction constant) has been deduced from mutual friction experiments in ^{4}He II [5], ^{3}He-B [6], and ^{3}He-A [7]; a full discussion of all these data is being published elsewhere [8]. The transverse force due to the normal fluid is usually quoted as a dimensionless parameter $d_\perp = -B/A$, and Wexler’s result $A = \rho_s \kappa$ is assumed. The usual analysis of these experiments assumes that the force on a vortex lattice is the sum of the forces on individual vortices. We believe that this assumption is justified because the collective modes of the vortex lattice are quite well understood [9] and are not excited in the mutual friction experiments. In ^{3}He-B, $d_\perp$ varies smoothly from approximately zero at the lowest temperatures to one at T_c, and also the nonzero $d_\perp$ has been shown to account for the observed shape of the free surface of the liquid in rotation experiments [10]. In ^{3}He-A, $d_\perp$ is indistinguishable from one over the whole temperature range where the A phase is stable in zero field. The measurements in ^{4}He are less accurate because of the low normal fluid viscosity, but it is clear that near the λ point $(1 - d_\perp) \propto (T_\lambda - T)^{1/3}$ [11].

We therefore believe that the correct conclusion to be drawn from Galilean invariance and the experimental facts is that there is an error in either Wexler’s argument or that of TAN. Wexler’s argument is based only on the quantization of circulation and thermodynamics [12] and seems particularly straightforward; moreover, as we have seen above, the result is not particularly controversial. We therefore suspect the TAN argument. The precise diagnosis of the problem should perhaps be left to professional theorists, but we would like to indicate an aspect of the argument that worries us. TAN use the device of a moving potential to move the vortex. If the situation being considered is one of thermal equilibrium it seems to us that the thermal excitations, including those in the vortex core, will move along with the potential and the vortex, so that no force proportional to $(\mathbf{v}_V - \mathbf{v}_n)$ will be registered. If, on the other hand, the situation envisaged is a steady state with continuous dissipation, it is not clear to us how the motion of the excitations is to be specified within the TAN argument.

H. E. Hall and J. R. Hook
Schuster Laboratory, University of Manchester, Manchester M13 9PL, United Kingdom

Received 18 August 1997 [S0031-9007(98)06081-5]
PACS numbers: 67.40.Vs, 47.37.+q, 67.57.Fg

[1] C. Wexler, Phys. Rev. Lett. **79**, 1321 (1997).
[2] G. E. Volovik, Phys. Rev. Lett. **77**, 4687 (1996).
[3] D. J. Thouless, Ping Ao, and Qian Niu, Phys. Rev. Lett. **76**, 3758 (1996).
[4] W. F. Vinen, Proc. R. Soc. London **A 260**, 218 (1961); S. C. Whitmore and W. Zimmermann Phys. Rev. **166**, 181 (1968); R. J. Zieve, Yu. M. Mukharsky, J. D. Close, J. C. Davis, and R. E. Packard, J. Low Temp. Phys. **91**, 315 (1993).
[5] C. F. Barenghi, R. J. Donnelly, and W. F. Vinen, J. Low Temp. Phys. **52**, 189 (1983).
[6] T. D. C. Bevan, A. J. Manninen, J. B. Cook, A. J. Armstrong, J. R. Hook, and H. E. Hall, Phys. Rev. Lett. **74**, 750 (1995); **74**, 3092(E) (1995).
[7] A. J. Manninen, T. D. C. Bevan, J. B. Cook, H. Alles, J. R. Hook, and H. E. Hall, Phys. Rev. Lett. **77**, 5086 (1996).
[8] T. D. C. Bevan, A. J. Manninen, J. B. Cook, H. Alles, J. R. Hook, and H. E. Hall, J. Low Temp. Phys. **109**, 423 (1997).
[9] E. B. Sonin, Rev. Mod. Phys. **59**, 87 (1987).
[10] H. Alles, J. P. Ruutu, A. V. Babkin, P. J. Hakonen, and E. B. Sonin, J. Low Temp. Phys. **102**, 411 (1996).
[11] P. Mathieu, A. Serra, and Y. Simon, Phys. Rev. **B 14**, 3753 (1976). Note that Eqs. (5) of Ref. [6] are valid for ^{4}He near the λ point.
[12] Wexler [1] states his argument in terms of Bose excitations, but it is readily adapted to the Fermi excitations of a Fermi superfluid.

