Experiments on the Dynamics of Vortices in Superfluid ^{4}He with No Normal Component

P. M. Walmsley[a], **A. A. Levchenko**[b], **and A. I. Golov**[a]

[a] *School of Physics and Astronomy, The University of Manchester, Manchester M13 9PL, UK*
[b] *Institute of Solid State Physics, Russian Academy of Sciences, Chernogolovka 142432, Russia*

Negative ions, probe particles of radius $\sim$ 10–20 Å, *can be injected into helium, manipulated and detected. They can be trapped by quantized vortices and hence used as vortex detectors. We show that by observing the change of the ion current caused by rotation of helium one can learn about the presence and dynamics of vortices even at very low temperatures.*

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1. INTRODUCTION

Recently, attention was attracted to the problem of turbulence in superfluid helium at very low temperatures such that there is no normal component left at all [1]. It is believed that in this limit the dynamics of turbulence may be different and more tractable than that in viscous classical liquids. To address the question experimentally, a vortex detection technique, operational at $T \ll 1$ K is required.

So far, two vortex detection techniques have dominated: the capture of injected ions by vortices and second sound attenuation due to vortices. The former was first used to prove that vortices are discrete continuous defects: first observation of a vortex tangle in 1960 [2], trapping of negative ions by a vortex array in 1962 [3], demonstration of the entry of vortices into a rotating container one by one [4]; visualization of vortex clusters [5], experimental proof that superfluid turbulence consists of a tangle of continuous defects – vortices [6], proof that remanent vortices always exist in ^{4}He [7]. Since then, the second sound technique has been mostly used (because of its excellent sensitivity and relative simplicity) to detect vortices [8] and turbulence in ^{4}He [9]. However, second sound does not propagate without a normal component, hence cannot be used for our purposes. A good review on the nature of vortices, turbulence and techniques can be found in the book by Donnelly [10].

There exist several other techniques: detection of pressure/temperature fluctuations associated with turbulence or the use of tracer particles. Attempts to use these techniques at $T < 1$ K are still in their infancy [11].

The first experiments to use ions for monitoring the dynamics of grid turbulence in ^{4}He at temperatures down to 20 mK was reported in 2000 by Davis et al.[12] They observed an apparent increase in the characteristic time of vortex decay with cooling from 200 to 100 mK, and then a temperature-independent process for 22–70 mK. However, to interpret this observation in terms of vortex density, one should know the efficiency of trapping of ions by vortex lines at these temperatures. The latter was unknown so far; our experiments were initially designed to fill this gap.

2. SPIN-UP IN SUPERFLUID ^{4}HE

Flow in a superfluid liquid is different from that in a classical liquid because the circulation is quantized in units of $\kappa = h/m$. In a simply-connected liquid rotational flow is not allowed (all the liquid can only move as a whole with the condensate). Rotational flow is however possible by introducing linear defects (vortex cores) of diameter $a \sim 1$ Å, along which the superfluidity is fully suppressed (hence, making the body of liquid essentially multiply-connected) and the circulation around each defect equals 1κ exactly. These linear defects are called *quantized vortices*. The instantaneous locations of their cores in space fully describe the flow everywhere in the container, the motion of the cores in response to the flow itself and frictional forces determines the dynamics of flow. These vortices are in a way similar to the eddies in classical liquids. However there are two important differences: they are all identical and they are topologically stable, i. e. they can neither terminate somewhere in the bulk of liquid, nor can they gradually appear or disappear (or even change circulation slightly). They can only extend all the way from wall to wall of a container or take a form of rings that do not have a beginning or end. The topology of vortex lines can change still because they can pass through each other and then *reconnect*. Two lines can swap their tails while keeping heads, lines can emit a ring, a ring can be absorbed by a line, etc. Reconnections are the major source of randomization in the system and account for a great deal of turbulent vortex dynamics.

When the container is stationary, the equilibrium state of liquid is the "Landau state", i. e. $\mathbf{v_s}(\mathbf{r}) = 0$ everywhere. In the absence of mutual friction at sufficiently low temperatures, this state can never be achieved in experiments (see the discussion in ref. [13]) because of very strong pinning of vortices on container walls (preserving cobwebs of metastable *remnant* vortices at all temperatures, especially in container corners), and very slow dynamics of slowing down the large-scale flow associated with *remanent* vortices. Awschalom and Schwarz observed the relaxation time of line density of 1–2 hours, and the final density of remnant vortices of some $L_R \leq 2\ln(D/a)/D^2$ in a container of size D [7].

The ever-present remnant vortices are actually quite useful, because they greatly reduce the hysteresis (i. e. critical velocity) related to the barrier for vortex production. When subjected to a pretty small flow, they start growing, reconnecting and thus multiplying, eventually assuring that the macroscopic superflow matches the velocity of container walls. The relevant *extrinsic* critical velocities are typically less than 1 cm/s (the bigger the container the smaller the extrinsic critical velocity), while the *intrinsic* critical velocity associated with the nucleation of nascent vortex rings is of order 50 m/s [15].

The equilibrium state of superfluid ^{4}He in a rotating container is solid body rotation, $\mathbf{v_s}(\mathbf{r}) = \Omega \times \mathbf{r}$, otherwise there would be a mismatch of velocities at the container surfaces. This is achieved by creating an array of parallel vortices [8] of uniform density $L = 2\Omega/\kappa$ (Feynman's rule) that mimics the velocity profile on average.

To transit from the stationary state to solid body rotation after sudden spin-up of the container from rest to certain Ω, the vortices have to move from the perimeter (where they are readily produced) to the center. The final state has a much smaller free energy in the rotating reference frame; without dissipating the kinetic energy of the macroscopic flow the transition to a regular array of vortices could never be achieved. Hence, the observation of the time required for the system to switch from the stationary state to solid body rotation is important in understanding the way vortices propagate in space and dissipate their energy. At high temperatures, where the mutual friction and viscosity of the normal component are present, the shear flow in the normal component will provide dissipation (see discussion in Section 6). Without the normal component, the main mechanisms are believed to be the excitation of large amplitude high-frequency Kelvin waves (helical waves of vortex cores) that emit phonons and friction experienced by pinned vortices sliding along the surface (that should emit phonons directly in the body of container walls). The former mechanism can work everywhere in the bulk liquid while the latter is only restricted to the boundary area.

Upon spin-up, we would expect a dense polarized tangle to be quickly created near the outer walls (the square cross-section of our container and grids in the middle of two opposite electrode plates should help speed up this process). The tangle is inhomogeneous: on its leading front there is a jump (discontinuity) of macroscopic velocity.

The evolution upon spin-down should proceed in a similar way: a dense tangle of turbulence (of opposite polarization) will be created at the walls and will start propagating inwards destroying the regular array on its way.

3. IONS IN HELIUM

The principles for using ions for detection of vorticity are as follows. Firstly inject ions and send them through the test volume. If there are vortices in the volume, some ions will be trapped and then restricted to move along vortices. The loss of ions and deflection of their current describe the density of vortex lines, their orientation and motion.

The injected ions are probe particles that can be pulled by external forces and are extremely useful for studies of excitations and vortices in liquid helium [14,10]. The negative ion is an electron self-localized in a bubble of radius 12–19 Å, while the positive ion is a cluster ion ("snowball") of radius 7–9 Å, dependent on pressure. By changing pressure and species, one can produce ion radii between 7 and 19 Å. Due to their larger size, negative ions interact with vortices stronger; hence we concentrate solely on them.

Ions are attracted to the core of quantized vortices due to the combination of the Bernoulli force and reduced condensation energy at the core; the binding energy for negative ions at $p = 0$ being of order 60 K [10]. Negative ions can either be injected into helium by field emission from a suitable metal, or as a product of ionizing helium atoms by α, β or γ particles.

At low temperature, an ion accelerates until it reaches either Landau velocity, $50 - 60$ m/s, or vortex nucleation velocity (see [15] and references therein). If the former is lower, the ion will continue to move at this velocity (they would cross our cell in 1 ms). However, if the latter is lower, a vortex ring will be nucleated that will capture the ion and this complex will be stable [16] unless the external field is too great.

The trapping diameter for bare ions was only measured down to $T =$ 0.8 K [17] and is of order 10^{-6} cm. On the other hand, the trapping diameter for charged rings was found to be of order their diameter [18], i.e. substantially larger than that of bare ions. Hence, it is much easier to detect vortices with ion-ring complexes; they were successfully used in past (see [19] and references therein). In this work we only use the rings at $T < 1$ K. The drawback of ion-ring complexes is their non-trivial dynamics (they are slow and very heavy), but this is tractable.

4. EXPERIMENTAL SETUP

Our aims were two-fold: to measure the cross-section of ion capture by vortex lines and to observe the vortex dynamics, both at sufficiently low temperatures $T < 200$ mK. In this paper we are describing the first successful experiments with liquid ^{4}He of natural purity at pressures $p = 1.2$ and 15 bar.

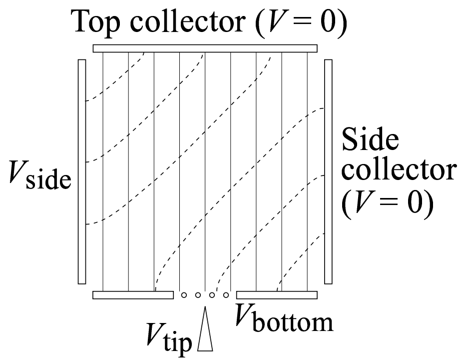

Fig. 1. Side view of the experimental cell. Dashed lines show the electric field lines for the configuration $V_{\rm bottom} = V_{\rm side}$ used in these experiments. Vertical solid lines represent rectilinear vortices in equilibrium at steady rotation.

The experimental cell is described in detail in our previous paper [20]. The ion drift space is confined by six square electrodes comprising a cube with a side of 4.5 cm. Ions were injected through a grid in the middle of the bottom electrode and their current could be detected by electrometers connected to the side and top collectors (see Fig. 1).

Ion injectors (a field-emitting metal tip) was manufactured by electro-etching tungsten wire in NaOH solution using the published recipe [21]. A current of some 10^{-11} A – 10^{-10} A was injected when the potential difference between the tip and the grid (i.e. bottom plate) reached $V_{\rm tip} - V_{\rm bottom}$ between -90 V and -110 V. In all measurements described in this paper, the electrode potentials were: $V_{\rm side} = V_{\rm bottom} = -90$ V; the side and top collectors at zero potential; two other electrodes (not seen in Fig. 1) were at -45 V. Throughout the paper, the current is quoted as the actual measured current multiplied by -1 (i.e. as if the electron charge would be positive).

A rotating dilution cryostat was used to produce an array of parallel vortex lines with inter-vortex spacing $\propto \Omega^{-1/2} \sim 0.2$ – 1 mm. A calibrated germanium resistance thermometer attached to the outer body of the cell was used to monitor the temperature.

5. EXPERIMENTAL RESULTS

At pressures 0–15 bar, especially in liquid ^{4}He with traces of ^{3}He impurities, at temperatures below 1 K a moving ion quickly nucleates a quantized vortex ring and then moves together with the ring [15]. The dynamics of such a complex (we will call them "charged vortex rings") is purely determined by the peculiar hydrodynamics of a ring, while the total energy acquired in an applied electric field is the electrostatic energy of the electron charge [16,10]. Fig. 2 shows the temperature dependence of the ion current arriving at the top and side collector when the cryostat is either stationary (open symbols) or rotating at $\Omega = 0.5$ rad/s (closed symbols). Free bare ions would follow the field lines and hence all of them would arrive in the middle of the side collector and none at the top collector. In the presence of an array of vertical rectilinear vortex lines caused by rotation, those ions that got trapped on these lines would slide up to the top collector being propelled by the vertical component of the electric field. And finally, sufficiently large (energetic) charged rings injected through the bottom grid downwards, thanks to their high inertia, would not be able to get deflected to the side collector even if not trapped on the rectilinear lines. See more about the dynamics of charged rings in this geometry in [20].

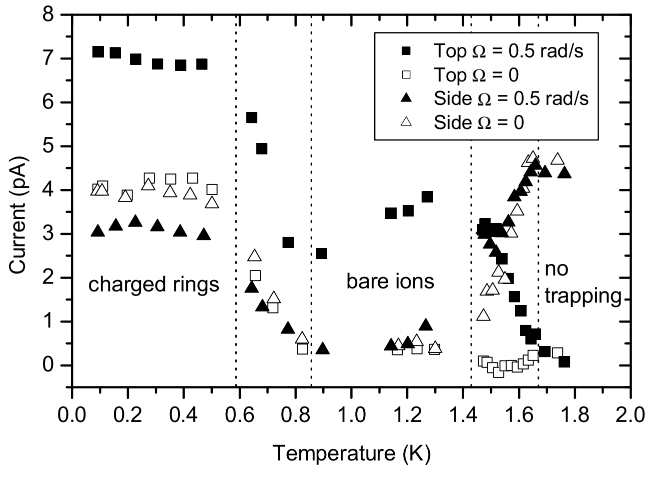

Fig. 2. Temperature dependence of the current to the top and side collectors with and without steady rotation. $p = 1.2$ bar.

In Fig. 2, one can see three distinct regimes:

- $T > 1.7$ K, all current arrives at the side collector independent of rotation (no trapping on vortices, free bare ions only);
- 0.9 K $< T <$ 1.4 K, with rotation, plenty of current is deflected to the top collector (bare ions dominate, they can get trapped on rectilinear vortices);
- $T < 0.6$ K, now even without rotation plenty of current arrives at the top collector carried by large charged vortex rings (charged rings dominate, they can hit a rectilinear vortex that results in the charge being transferred onto the latter).

All these observations agree with the existing knowledge of the ion behavior, outlined above. We see that at $T < 0.6$ K the majority of injected ions end up as charged vortex rings. At the voltages used in our experiment, ~ 100 V, the rings grow to some 10^{-4} cm and cross the cell in about 1 s. At the moment, we cannot say where exactly they nucleate (i. e. whether before or after crossing the grid and entering the main volume) and how broad the distribution of their energies (radii) is. However, our most recent experiments with another field emission tip (which had a higher threshold voltage for electron emission of ≈ 230 V) showed that the majority of charged rings were nucleated sufficiently close to the tip [20] and they enter the main volume with a pretty well-defined energy (radius) [22].

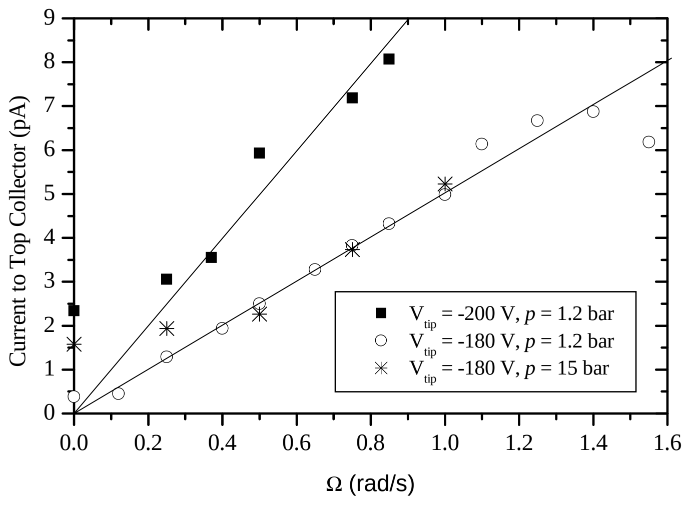

Fig. 3. Current to the top collector at steady rotation at different angular velocities; $T < 100$ mK. Straight lines guide the eye.

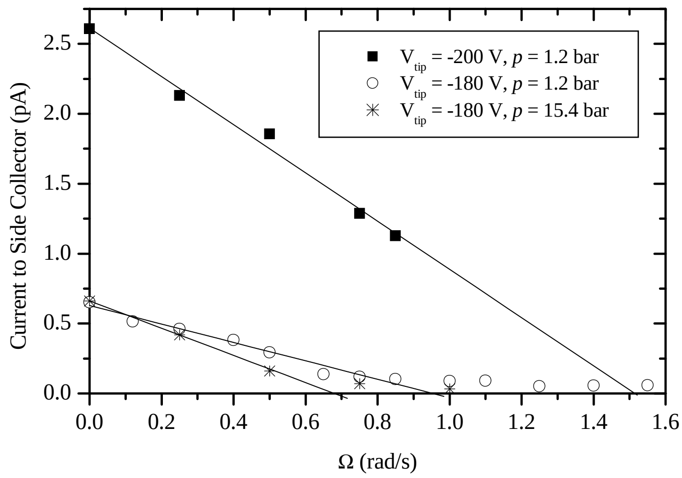

Fig. 4. Current to the side collector at steady rotation at different angular velocities; $T < 100$ mK. Straight lines extrapolate the linear dependence at small Ω.

For our domain of interest, $T << 0.5$ K, we are thus left with the charged rings as the potential detectors of vortices. Schwarz and Donnelly [18] have found that they interact with rectilinear vortex lines very effectively, with the diameter for charge transfer approximately equal to the ring diameter which is of order 1 μm in our case. In Fig. 3 and Fig. 4 we are plotting the dependence of the current reaching the side and top collectors at different angular velocities of rotation. With increasing Ω, more current gets channelled to the top collector (except at the lowest values of Ω which is probably the effect of remnant vortices) and less arrives at the side electrode. The behaviors at different pressures and magnitudes of injected current are very similar.

To describe the depletion of the current to the side collector, an exponential dependence of I/I_0 on Ω is expected:

$$I = I_0 \exp(-\Omega/\Omega^*), \qquad (1)$$

where $\Omega^* = \kappa/2X\sigma$ is the position of the extrapolated intercept of the straight lines with the horizontal axis, σ is the trapping diameter for the typical angle of motion of the ring relative to the vortex line, $X \approx 5$ cm is the distance travelled by a not-trapped charged ring between the injector and collector. All values of the intercepts are about $\Omega^* \sim 1$ rad/s. This gives $\sigma \sim 2 \times 10^{-4}$ cm, that is similar to the values of the trapping diameter for charged rings at higher temperatures [18]. This observation implies that even at $T < 0.1$ K we are perfectly capable of detecting even small densities of vortex lines by their effect (i.e. trapping) on the injected charged vortex rings. Hence, we can proceed to our second objective to try to observe the time dependence of the vortex density after starting or stopping rotation at low temperatures.

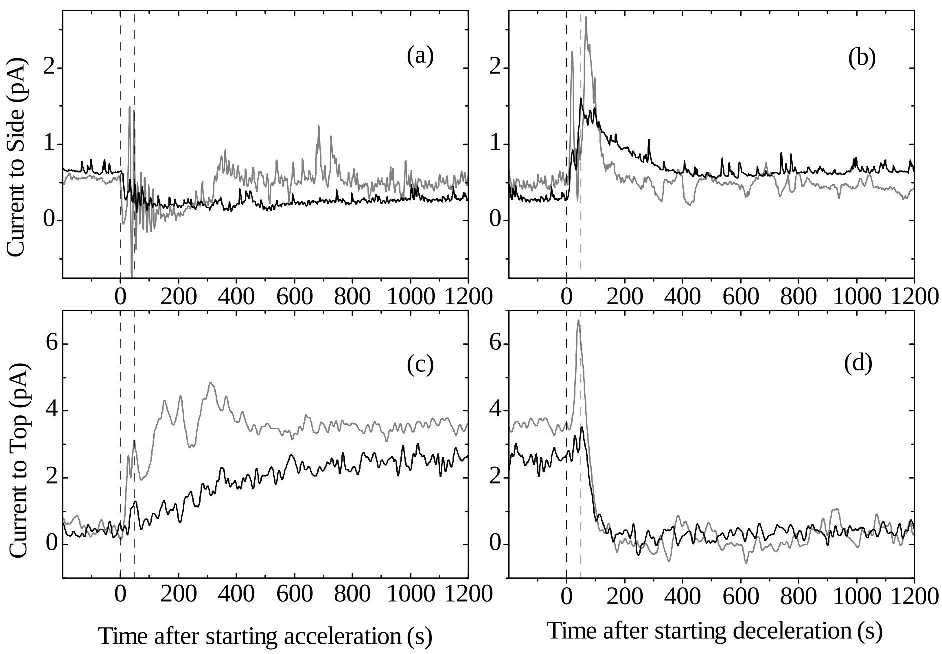

Fig. 5. Transients of the ion current to the side (a, b) and top (c, d) collectors after starting (a, c) rotation at $t = 0$ and reaching $\Omega = 0.5$ rad/s in 50 s or stopping (b, d) rotation at $\Omega = 0.5$ rad/s at $t = 0$ and reaching $\Omega = 0$ in 50 s (vertical dashed lines indicate the 50-second intervals of uniform acceleration/deceleration). Grey line: $T = 1.21$ K, $V_{\rm tip} = -200$ V; black line: $T = 0.06$ K, $V_{\rm tip} = -180$ V. $p = 1.2$ bar.

Figures 5 (a) and (c) show, for temperatures 1.2 K and 60 mK, the transients of the current to the top and side collectors upon spinning the cryostat up to $\Omega = 0.5$ rad/s within 50 seconds. Figures 5 (b) and (d) show the outcomes of stopping the rotation. The main result here is that, although the transients at high and low temperatures look markedly different, the time needed to reach a quasi-steady current is not very different at high and low temperatures. Namely, it is about 500 s for starting and 200 s for stopping rotation at $T = 1.2$ K, and about 800–1000 s for starting and 400 s for stoping rotation at $T = 0.06$ K.

The transients look pretty noisy. However, some part of the fine structure is not random electronics noise but the fluctuations of the current of ions trapped on vortices (including the displacement current, i.e. the capacitive pick-up) caused by the large-scale motion inside the experimental cell. It is interesting to see that the transients at low temperature actually look more smooth and predictable than those for high temperatures with their numerous spikes and other structures.

6. DISCUSSION

At temperatures above 1 K, the Hall-Vinen-Bakarevich-Khalatnikov (HVBK) equations can be used to calculate the dynamics of helium. At these temperatures, thanks to substantial mutual fiction, the large-scale velocity fields of the superfluid and normal components are essentially locked together. The inertia is given by the total density of liquid ρ while the damping is provided by the viscosity of the normal component $\eta_{\rm n}$. Hence, the characteristic times should scale as

$$\tau_0 = R^2 \rho \eta_{\rm n}^{-1}, \tag{2}$$

where R is the length scale. Using $R = 2.25$ cm and $\eta_{\rm n}\rho^{-1} = 10^{-4}$ cm$^2 s^{-1}$, we thus get $\tau_0 \sim 5 \cdot 10^4$ s. Idowu, Henderson and Samuels [23] have recently addressed the spin-up and decay of vorticity in such coupled flow using HVBK equations. They considered a cylindrical volume and symmetric solutions with only radial dependence (i.e. no large-scale turbulent motion was allowed). They obtained the typical time for vorticity to relax of some $0.01\tau_0 \sim 500$ s, which is close to our observations.

From the observed transients, it seems that at low temperatures the propagation (or decay) of vorticity in the bulk of the cell is progressing in a more orderly fashion than at high temperature. We can only speculate that the fine structure of the high-temperature transients is related to the large-scale 3-d motion in the turbulent normal component in the cubic cell. That the normal component should be turbulent is clear from the high value of its Reynolds number at our typical $\Omega = 1$ rad/s, ${\rm Re_n} = \Omega R^2 \rho/\eta_{\rm n} = 50,000$.

Now let us attempt to interpret the transients for $T = 0.06$ K. We should bear in mind that any type of produced vorticity (either regular array that quickly removes the trapped charge or turbulent tangle that creates space charge of trapped ions which block the current) will deplete the ion current to the side collector, while only a regular array of vertical rectilinear vortex lines will be able to effectively channel ions up to the top collector. Upon spin-up we can see that the current to the side collector basically plummets within first 100 s (Fig. 5, (a)), while the current to the top collector keeps crawling up for about 1000 s (Fig. 5, (c)). The former time, 100 s, corresponds to the initial building up of a tangle of vorticity near the perimeter walls, while the latter, 1000 s, signals establishing a coherent structure of rectilinear vortices reaching the top collector (i.e. attainment of the solid body rotation state). Upon spin-down, it takes some 500 s for the current to the side collector to relax (Fig. 5, (b)), while the current to the top collector settles in about 150 s (Fig. 5, (d)). Again, we infer that the time of 150 s corresponds to the destroying of the regular vortex array, while 500 s is required for the major density of the turbulent tangle to decay (thus the superfluid approaching "mainly stationary" state with only remnant vortices). To make interpretation less speculative, we are currently attempting to formulate a theoretical framework to describe the propagation of the front of vorticity at low temperatures; this will be addressed in forthcoming publications. Numerical simulations of type presented in [24,25] might be required to shed light on the internal dynamics of the vortices upon spin-up and spin-down.

It is worth mentioning that with just one collector, we would be unable to distinguish the effect of a regular vortex array from that of a turbulent tangle. Fortunately, we could monitor the current flow to at least three different electrodes simultaneously. This will be tried in our further experiments.

7. CONCLUSION

Our first attempt of detecting the change in vortex density upon starting and stopping rotation at $T < 0.5$ K was obviously a success: we were able to detect vortices by their interaction with injected negative ions down to 30 mK. So far only a direct current of charged vortex rings (not bare negative ions) was used for this purpose, but these rings turned out to be quite convenient.

The dynamics of spin-up and spin-down were probed at various temperatures. We found that at $T < 100$ mK the flow relaxes in some 600 s, i.e. almost as quickly as at $T > 1$ K. However the current transients are markedly different.

The used DC current technique is known to have drawbacks such as its sensitivity to the accumulated surface charge and build-up of space charge on vortices. To improve the quality of data and to make their interpretation less ambiguous, we are going to try a pulse technique in further experiments.

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