

# Interaction-Driven Spectrum Reconstruction in Bilayer Graphene

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The nematic phase transition in electronic liquids, driven by Coulomb interactions, represents a new class of strongly correlated electronic ground states. We studied suspended samples of bilayer graphene, annealed so that it achieves very high quasiparticle mobilities (greater than  $10^6$  square centimeters per volt-second). Bilayer graphene is a truly two-dimensional material with complex chiral electronic spectra, and the high quality of our samples allowed us to observe strong spectrum reconstructions and electron topological transitions that can be attributed to a nematic phase transition and a decrease in rotational symmetry. These results are especially surprising because no interaction effects have been observed so far in bilayer graphene in the absence of an applied magnetic field.

Although many consequences of the chiral nature of the quasiparticles in mono- (1–3) and bilayer (4–5) graphene have already been demonstrated (3), sample inhomogeneities and broadening of the energy levels (due to a finite lifetime of the charge carriers) prohibited direct access to the intriguing regime of zero electron and hole concentrations. The latter regime has attracted a large number of possible scenarios for the particular arrangement of the electron ground state. For bilayer graphene, the single-electron picture (5) (where the parabolic spectrum is replaced by four Dirac cones at low energies) is challenged by several many-body phase transition scenarios: ferromagnetic (6), ferroelectric (7, 8), and nematic (9, 10). The latter resembles the transitions in correlated anisotropic electron fluids (11–13) that have been observed in several systems, including electrons in high Landau levels in two-dimensional electron gas (14–16), multilayered complex oxides (17, 18), iron pnictides (19), and strontium ruthenate (20).

Recently, the required access to the low-energy physics in bilayer crystals has been enabled by improvements in the quality of samples created by removing the underlying substrate (21–23). Free-standing graphene devices allow a 100-fold increase in the mobility (above  $10^6$  cm<sup>2</sup>/V·s) (24) after annealing. We studied the low-energy electronic properties of such high-quality, free-standing bilayer graphene devices and demonstrate that electron-electron interaction effects play a dominant role at low energies, even at low magnetic fields strengths  $B$ . Our samples were prepared on Si/SiO<sub>2</sub> substrate (300 nm of SiO<sub>2</sub>) by micro-mechanical cleavage (1). Narrow (2 to 4 μm) ribbons were selected, and Cr/Au (5 nm/150 nm) contacts were deposited by means of electron-

beam or laser-writer lithography and electron-beam evaporation to create two-probe devices. Wet etching in buffered hydrofluoric acid was used to remove about 150 nm of SiO<sub>2</sub> substrate to form free-standing bilayer graphene devices (Fig. 1C).

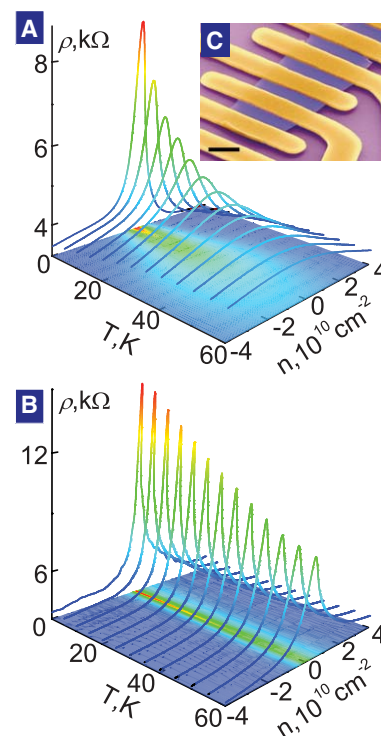
As prepared, such samples were p-doped to about several  $10^{12}$  cm<sup>-2</sup> and exhibited mobility traditionally expected for samples on a substrate (25) (~5000 cm<sup>2</sup>/V·s). We used current annealing (22) to remove such unintentional doping and increase the mobility (typically 0.5 to 1 mA/μm current densities were used). After annealing, our devices exhibited a very sharp resistance peak situated almost exactly at zero gate voltage (within  $10^8$  cm<sup>-2</sup> in terms of residual doping; Fig. 1, A and B). Typical quantum mobilities achieved were within  $\mu_q = 0.5 \times 10^6$  to  $1.5 \times 10^6$  cm<sup>2</sup>/V·s (estimated from the onset of quantum oscillations in magnetic field  $B_0$  as  $\mu_q B_0 = 1$ ). Transport mobilities were more difficult to estimate because of the influence of the contacts in the two-probe geometry and nearly ballistic transport in our high-quality samples. However, a rough estimate made from the slope of the conductivity curve (field effect mobility:  $\mu = 1/e \cdot d\sigma/dn$ , where  $\sigma$  is conductivity) gave similar numbers (within 30%).

Our experiments on electronic transport in free-standing bilayer graphene in zero and finite magnetic fields show that at energies below 5 meV, the parabolic dispersion [as usually observed at high energies (4)] is replaced, through an electron topological transition, by several pockets with a linear spectrum. Also, our magnetic field measurements show the presence of only two Dirac cones around each K point in the Brillouin zone, in contrast with the single-particle theory (5) that predicts four Dirac cones around each K point at energies below 1 meV. We interpret the results of our experiments as an indication for interaction-driven phase transition (9).

All of the samples in our study had practically zero doping (within  $10^8$  cm<sup>-2</sup>), which allowed us to estimate the possible energy gap that would arise from asymmetry between the layers. This gap is lower than 0.1 meV, which is an order of magnitude smaller than the effects we describe. The absence of the gap is also manifested in the relatively weak temperature dependence of the

resistivity peak (conductivity minimum) down to the lowest temperatures measured ( $T = 0.25$  K) (Figs. 1A and 2A). At low temperatures, the conductivity saturates at a finite value of about  $20 e^2/\pi h$  ( $h$ , Planck's constant) (see fig. S1 for details), which is substantially greater than the value expected for ballistic monolayer graphene (26, 27) ( $4e^2/\pi h$ ) or bilayer graphene with a parabolic dispersion relation (28) ( $8e^2/\pi h$ ). The measured minimum conductivity is slightly lower than  $24e^2/\pi h$  predicted (29) for bilayer spectrum transformed by the electron topological transition at low Fermi energies (5) (Fig. 2B).

The broadening and the amplitude of the conductivity minimum were linear functions of temperature for  $T > 20$  K. Broadening of the conductivity minimum effectively yielded the same information as the amplitude, but without the ambiguity that arises from the contact resistance and the quantum minimum conductivity (see SOM text for the definition of broadening we use here). Such behavior is expected for the parabolic dispersion relation, which guarantees a constant density of states (conductivity:  $\sigma_{\min} = e\mu[D(E)[f(E,T) - f(E,T=0)]]dE = 4\ln(2)e\mu mT/\pi\hbar^2$ , where  $D(E) = \text{constant}$  is the density of states,  $f(E,T)$  is the Fermi-Dirac distribution function, and  $m$  is the effective mass, and we assumed that the mobility is independent of the temperature and the position of the



**Fig. 1.** Temperature dependence of the resistance in graphene and bilayer graphene. (A) Temperature dependence of two-terminal resistance of bilayer graphene at zero magnetic field. (B) Temperature dependence of two-probe resistance of monolayer graphene at zero magnetic field. (C) A scanning electron micrograph of one of our suspended devices.

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Fermi level). The slope of the temperature dependence yielded  $m \approx (0.029 \pm 0.003)m_0$  (Fig. 2B) (here  $m_0$  is the free electron mass). Note that according to the same arguments, the conductivity for the monolayer graphene and broadening of the conductivity minimum increased much more slowly and [because  $D(E) \propto |E|$  for the linear spectrum] is a parabolic function of  $T$  (Fig. 1B and Fig. 2, B and C).

For  $T < 20$  K, the temperature dependence of the width of the conductivity minimum for bilayer graphene was substantially weaker than what is expected for the parabolic dispersion relation (the amplitude of the conductivity minimum yields a similar conclusion). A slow  $T$  dependence hints that the density of states at low energy is suppressed, although it remains finite. Such behavior has been predicted (5) for the low-energy part of the bilayer spectrum, below the saddle-point  $E^*$  in the electron dispersion, where the high-energy parabolic spectrum is replaced by four Dirac minicones (see inset to Fig. 2B); the value of  $E^*$  is determined by the amplitude of the next-nearest neighbor interlayer hopping  $\gamma_3$ , and the number of pockets by the trigonal symmetry of this parameter.

A change in the band structure topology, which consists of a single-connected isoenergetic line splitting into disconnected parts, is known as the electron topological or Lifshitz transition (30). In bilayer graphene, such a transition is expected to occur at  $E^* \sim 1$  meV, yielding the temperature dependence presented in Fig. 2A (green curve). Although such theoretical prediction qualitatively captures the general trend (transition from parabolic to linear temperature dependence), it strongly underestimates the value of  $E^*$ . It requires  $E^*$  on the order of 6 meV to fit our experimental data (Fig. 2A, red curve; see below for the details of the model used).

To study the electron topological transition in more detail and search for evidence of many-body effects, we measured the transport properties of such high-quality bilayer samples in an applied magnetic field. Our measurements are summarized in Fig. 3, where the resistivity is plotted as a function of magnetic field and gate voltage. We observed nonmonotonic changes in resistivity at filling factor  $\nu = 0$  (Fig. 3B). The sign of magnetoresistance changed twice: It was positive at low ( $< 0.1$  T) and high ( $> 0.3$  T) magnetic fields and negative in the intermediate re-

gion. Generally, higher-quality samples exhibited these features at lower fields. We interpret the divergence of the resistance at high fields ( $B > 0.3$  T) as the gap opening at  $\nu = 0$ , similar to that observed previously (23, 31). The negative magnetoresistance at  $0.1 \text{ T} < B < 0.3 \text{ T}$  has a resonant character—a narrow cusp appears in resistance in a narrow area of concentrations ( $\pm 2 \times 10^9 \text{ cm}^{-2}$ ) (Fig. 3C and figs. S2 and S3), an effect that we cannot explain at present. Data for a device with different mobility is shown in fig. S2.

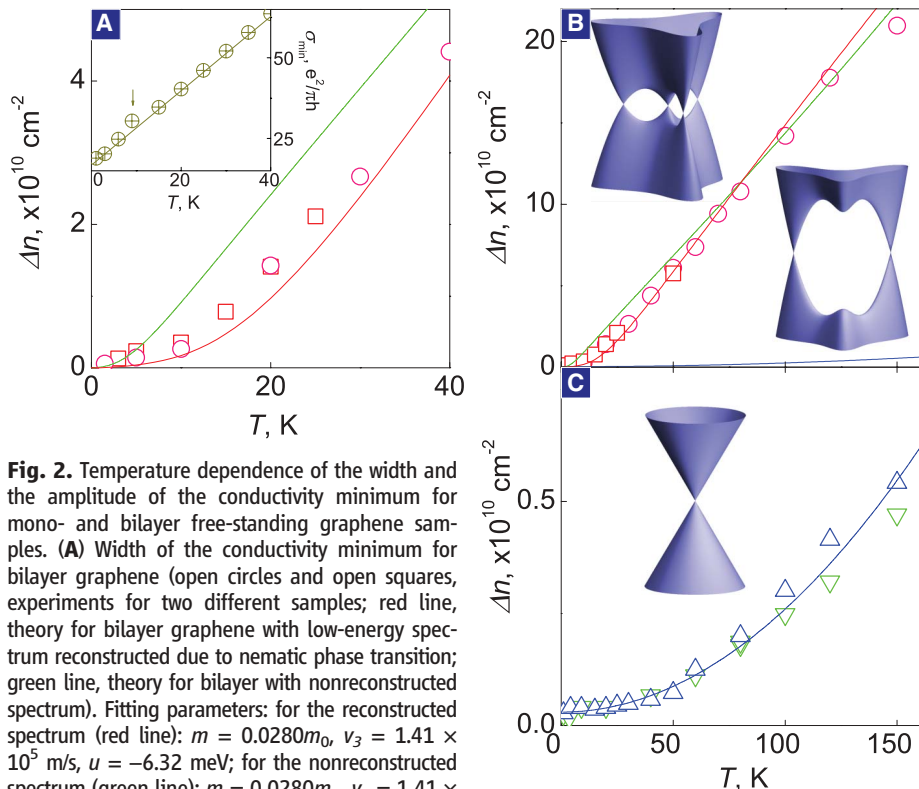
We also observed that the cyclotron gaps at filling factor  $\nu = \pm 4$  (the filling factor is determined from the carrier concentration  $n$  as  $\nu = nh/eB$ ) were much more robust than the gaps at other filling factors and could be observed at magnetic fields as low as 100 G [the other filling factors are observed only at  $B > 0.4$  T even for the highest-quality samples, which is consistent with the results obtained by other groups (31)]. Figure 4 summarizes our measurements of the cyclotron gaps at various filling factors (obtained from fitting of the temperature dependence of oscillations presented in Fig. 3C with the Lifshitz-Kosevich formula). The cyclotron gaps at filling factors  $\pm 8, \pm 12, \pm 16, \pm 20$ , and  $\pm 24$  showed a linear dependence on magnetic field (Fig. 4). We extracted the corresponding cyclotron mass, which appears to be  $m = (0.028 \pm 0.002)m_0$ —close to the previously reported value for nonsuspended samples (32). At the same time, the gap for  $\nu = \pm 4$  rose quickly for  $0 \text{ T} < B < 0.02 \text{ T}$  and then stayed practically constant at  $E_L \approx 5.5 \text{ meV}$  for  $0.02 \text{ T} < B < 0.8 \text{ T}$ , before increasing linearly with a slope  $\sim 40\%$  greater than that for other filling factors.

The reported observation cannot be explained within the framework of parabolic bands, which predicts roughly equidistant Landau levels. It also contradicts the more elaborate theory (5) (beyond the nearest-neighbor hopping approximation), which predicts four Dirac minicones below the electron topological transition energy in the vicinity of both K and K' points. Similar to graphene, each of such Dirac pockets would result in a doubly degenerate Landau level, so the total degeneracy of the zero Landau level is 16, and the largest cyclotron gap should correspond to filling factor  $\pm 8$ , instead of  $\pm 4$  as observed in our experiments.

To explain the observed eight-fold degeneracy of the lowest Landau level, we need to assume that the rotational symmetry of the system is lowered by, e.g., the nematic phase transition (9, 10), which results in the following Hamiltonian for the low-energy electrons,

$$H = -\frac{1}{2m} \begin{pmatrix} 0 & (\pi^\dagger)^2 \\ \pi^2 & 0 \end{pmatrix} + \xi v_3 \begin{pmatrix} 0 & \pi \\ \pi^\dagger & 0 \end{pmatrix} + \begin{pmatrix} 0 & u \\ u & 0 \end{pmatrix}$$

and reduces the number of low-energy Dirac minicones from four to two near each of the



**Fig. 2.** Temperature dependence of the width and amplitude of the conductivity minimum for mono- and bilayer free-standing graphene samples. **(A)** Width of the conductivity minimum for bilayer graphene (open circles and open squares, experiments for two different samples; red line, theory for bilayer graphene with low-energy spectrum reconstructed due to nematic phase transition; green line, theory for bilayer with nonreconstructed spectrum). Fitting parameters: for the reconstructed spectrum (red line):  $m = 0.0280m_0$ ,  $v_3 = 1.41 \times 10^5 \text{ m/s}$ ,  $u = -6.32 \text{ meV}$ ; for the nonreconstructed spectrum (green line):  $m = 0.0280m_0$ ,  $v_3 = 1.41 \times 10^5 \text{ m/s}$ ,  $u = 0$ . (Inset) Amplitude of the conductivity minimum of bilayer graphene (yellow crossed circles, experiment; yellow solid line, a guide to the eye). Note the deviation from the straight line below 10 K (marked by arrow). **(B)** The broadening of the conductivity minimum for bilayer samples [circles, squares, and red and green lines are the same as in (A) and for monolayer graphene (blue line, theory)]. (Inset) Left: Low-energy electronic spectrum as expected in the single-electron approximation; right: bilayer graphene low-energy electronic spectrum, reconstructed due to nematic phase transition. **(C)** The broadening of the conductivity minimum for monolayer graphene [blue and green triangles: experimental points for two different samples; blue line: theory, same as in (B)]. (Inset) Low-energy electronic spectrum for monolayer graphene.

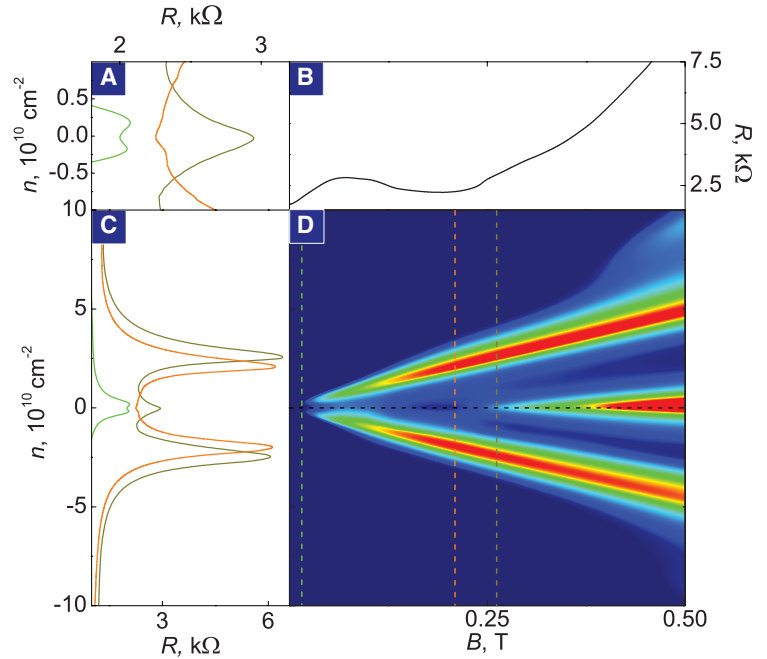
two K points (Fig. 4A). In the above Hamiltonian,  $\xi$  differentiates between the two valleys,  $v_3$  is the velocity associated with the interlayer skew hopping (5), and  $u$  is the order parameter formed upon nematic phase transition. This transition is driven by Coulomb interaction between electrons (the strength of which is partly reduced by screening), and it has been established by the renormalization-group study of bilayer parameters and the strength of the interaction

between all types of symmetry-breaking fluctuation in the electronic liquid in bilayer graphene (9, 10). We could fit our experimental results for the cyclotron gaps rather accurately (Fig. 4, B and C). The range of parameters that fit our results was very narrow, and the same set of parameters fit our temperature broadening data at zero magnetic field (Fig. 2, A and B).

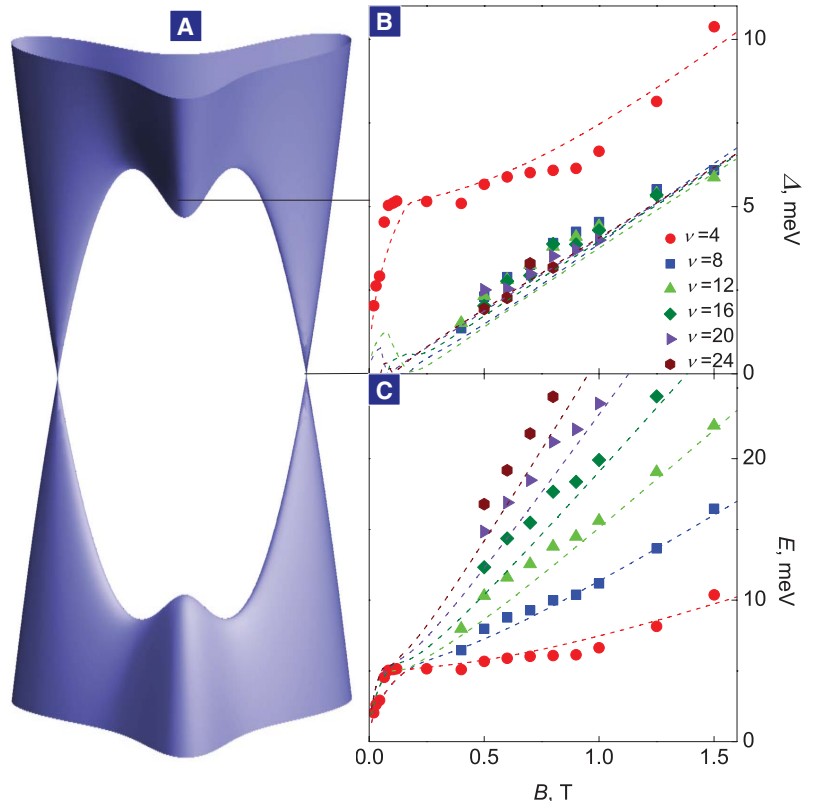
Lowering of the symmetry can be expected, either in the case of the spontaneous symmetry

breaking caused by many-body effects or in the case of mechanical deformation of our devices, either by uniaxial strain of the whole crystal (33) or by interlayer shear shift (34) caused by their preparation or handling history. However, the latter scenario can be ruled out because the strain required to explain the results observed in our work could be estimated (33) to be on the order of 1%, whereas the typical strain found in similar devices (prepared and handled in the same

**Fig. 3.** Free-standing bilayer samples in magnetic field. **(A)** Resistance of suspended bilayer graphene as a function of gate voltage (recalculated to carrier concentration) for  $B = 0.01$  T (green),  $B = 0.2$  T (orange), and  $B = 0.26$  T (dark yellow). **(B)** Resistance of suspended bilayer graphene at  $\nu = 0$  as a function of magnetic field. **(C)** Same as (A) for a greater span of the carrier concentration. **(D)** Contour map of resistance of free-standing bilayer graphene device as a function of carrier concentration and magnetic field. The resistance spans from 2.1 kilohm (blue) to 6 kilohm (red). The three vertical lines correspond to the positions of the  $B = 0.01$  T (green),  $B = 0.2$  T (orange), and  $B = 0.26$  T (dark yellow) in (A) and (C).



**Fig. 4.** **(A)** Low-energy electron spectrum for graphene bilayer, reconstructed due to nematic phase transition. **(B)** Cyclotron gaps for different filling factors. Symbols (experiment) and curves (theory) of the same color correspond to the same filling factor (fitting parameters:  $m = 0.0280m_0$ ,  $v_3 = 1.41 \times 10^5$  m/s,  $u = -6.32$  meV). **(C)** The position of the Landau levels as a function of magnetic field, recalculated from (B). Symbols (experiment) and curves (theory) of the same color correspond to the same filling factor as in (B).



way) was measured (24) and was  $<0.1\%$ . Moreover, in the case of externally induced strain, the direction of mechanical deformations would be related to the contact geometry, not to the crystallographic orientation, and would vary from sample to sample. Instead, we find the same feature in four different devices, which would require a rather narrow range of parameters (size and orientation of the principal axes of strain tensor) to be repeated precisely in different samples. We also have considered whether our results can be explained in terms of a spontaneous gap opening in the bilayer spectrum (35). In this case, the transport at the neutrality point (with or without magnetic field) would be dominated by a network of one-dimensional channels between the domains of gaps with alternating signs. This would result in a minimum conductivity value that depends strongly on the strain prehistory, in contrast to values reproducibly observed in our experiments. Also, this model would not likely produce a drop in the resistance at  $\nu = 0$ , as observed in our experiments. These differences suggest that the observed reconstruction of the spectrum is the result of an intrinsic modification of the electronic system; in particular, it can be caused by the recently predicted “nematic” phase transition (9, 10) in bilayer graphene.

## References and Notes

1. K. S. Novoselov *et al.*, *Science* **306**, 666 (2004).
2. A. K. Geim, K. S. Novoselov, *Nat. Mater.* **6**, 183 (2007).
3. A. H. Castro Neto, F. Guinea, N. M. R. Peres, K. S. Novoselov, A. K. Geim, *Rev. Mod. Phys.* **81**, 109 (2009).
4. K. S. Novoselov *et al.*, *Nat. Phys.* **2**, 177 (2006).
5. E. McCann, V. I. Fal'ko, *Phys. Rev. Lett.* **96**, 086805 (2006).
6. E. V. Castro, N. M. R. Peres, T. Stauber, N. A. P. Silva, *Phys. Rev. Lett.* **100**, 186803 (2008).
7. R. Nandkishore, L. Levitov, *Phys. Rev. Lett.* **104**, 156803 (2010).
8. F. Zhang, H. Min, M. Polini, A. H. MacDonald, *Phys. Rev. B* **81**, 041402 (2010).
9. Y. Lemonik, I. L. Aleiner, C. Toke, V. I. Fal'ko, *Phys. Rev. B* **82**, 201408 (2010).
10. O. Vafek, K. Yang, *Phys. Rev. B* **81**, 041401 (2010).
11. V. Oganesyan, S. A. Kivelson, E. Fradkin, *Phys. Rev. B* **64**, 195109 (2001).
12. E. Fradkin, S. A. Kivelson, V. Oganesyan, *Science* **315**, 196 (2007).
13. E. Fradkin, S. A. Kivelson, M. J. Lawler, J. P. Eisenstein, A. P. Mackenzie, *Annu. Rev. Condens. Matter Phys.* **1**, 153 (2010).
14. M. P. Lilly, K. B. Cooper, J. P. Eisenstein, L. N. Pfeiffer, K. W. West, *Phys. Rev. Lett.* **82**, 394 (1999).
15. M. P. Lilly, K. B. Cooper, J. P. Eisenstein, L. N. Pfeiffer, K. W. West, *Phys. Rev. Lett.* **83**, 824 (1999).
16. R. R. Du *et al.*, *Solid State Commun.* **109**, 389 (1999).
17. Y. Ando, K. Segawa, S. Komiya, A. N. Lavrov, *Phys. Rev. Lett.* **88**, 137005 (2002).
18. V. Hinkov *et al.*, *Science* **319**, 597 (2008).
19. T. M. Chuang *et al.*, *Science* **327**, 181 (2010).
20. R. A. Borzi *et al.*, *Science* **315**, 214 (2007).
21. X. Du, I. Skachko, A. Barker, E. Y. Andrei, *Nat. Nanotechnol.* **3**, 491 (2008).
22. K. I. Bolotin *et al.*, *Solid State Commun.* **146**, 351 (2008).
23. B. E. Feldman, J. Martin, A. Yacoby, *Nat. Phys.* **5**, 889 (2009).
24. E. V. Castro *et al.*, *Phys. Rev. Lett.* **105**, 266601 (2010).
25. L. A. Ponomarenko *et al.*, *Phys. Rev. Lett.* **102**, 206603 (2009).
26. See supporting material on Science Online.
27. M. I. Katsnelson, *Eur. Phys. J. B* **51**, 157 (2006).
28. I. Snyman, C. W. J. Beenakker, *Phys. Rev. B* **75**, 045322 (2007).
29. J. Cserti, A. Csordás, G. Dávid, *Phys. Rev. Lett.* **99**, 066802 (2007).
30. I. M. Lifshitz, *Sov. Phys. JETP* **11**, 1130 (1960).
31. R. T. Weitz, M. T. Allen, B. E. Feldman, J. Martin, A. Yacoby, *Science* **330**, 812 (2010).
32. E. V. Castro *et al.*, *Phys. Rev. Lett.* **99**, 216802 (2007).
33. M. Mucha-Kruczyński, I. L. Aleiner, V. I. Fal'ko, *Phys. Rev. B* **84**, 041404 (2011).
34. Y.-W. Son, S.-M. Choi, Y. P. Hong, S. Woo, S.-H. Jhi, <http://arxiv.org/abs/1012.0643v1> (2010).
35. R. Nandkishore, L. Levitov, *Phys. Rev. B* **82**, 115124 (2010).

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## Supporting Online Material

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Figs. S1 to S3

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# Interaction-Driven Spectrum Reconstruction in Bilayer Graphene

(Supplementary Information)

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## Thermal Broadening of the Conductivity Minimum

Neglecting electron-electron interaction, one can write the static conductivity in the form:

$$\sigma(T, \xi) = \int_{-\infty}^{\infty} K(E) \left( -\frac{\partial f(E, T, \xi)}{\partial E} \right) dE, \quad (1)$$

where  $f(E, T, \xi)$  is the Fermi distribution function

$$f(E, T, \xi) = \frac{1}{1 + e^{(E - \xi)/T}}, \quad (2)$$

$\xi$  is a chemical potential and  $K(E)$  is an unknown function of the energy only to be defined[1]. The expression (1) is valid for any elastic scattering mechanisms, it follows from the general Kubo formula under the assumption that the Hamiltonian and the current density are sums of single-particle operators. It can be rewritten as

$$\sigma(T, \xi) = K(\xi) + \int_0^{\infty} \frac{dx}{1 + e^x} \frac{\partial}{\partial x} [K(\xi + xT) + K(\xi - xT)] \quad (3)$$

The experimentally measured dependence is  $\sigma(T) = \sigma(T, \xi(T))$ , where the temperature dependence of the chemical potential can be found from the condition, that the concentration  $n$  persists. The latter can be computed as  $n_e - n_h$ , where

$$n_e = \int_0^{\infty} N(E) f(E, T, \xi) dE, \quad n_h = \int_{-\infty}^0 N(E) [1 - f(E, T, \xi)] dE \quad (4)$$

are total numbers of electrons and holes respectively and  $N(E)$  is the density of states.

We find

$$n(T, \xi) = \int_0^{\infty} N(E) \left( \frac{1}{1 + e^{(E - \xi)/T}} - \frac{1}{1 + e^{(E + \xi)/T}} \right) dE, \quad (5)$$

where we assumed that  $N(E)$  is even (electron-hole symmetry). Thus

$$n(T, \xi(T)) = \int_0^{\infty} N(E) \left( \frac{1}{1 + e^{(E - \xi(T))/T}} - \frac{1}{1 + e^{(E + \xi(T))/T}} \right) dE = n = \text{const.} \quad (6)$$

At zero temperature  $\sigma(T = 0, \xi) = K(\xi)$ . On the other hand, *experimentally*,

$\sigma(T = 0, \xi_0) \propto |n(T = 0, \xi_0)|$ , where  $\xi_0 = \xi(0)$ . This gives

$$\sigma(T=0, \xi_0) = K(\xi_0) = \beta |n(T=0, \xi_0)| = \beta \int_0^{|\xi_0|} N(E) dE, \quad (7)$$

where  $\beta$  does not depend on  $\xi_0$ . Equation (7) defines  $K(\xi)$ . Substituting (7) into (3) we obtain for the conductivity

$$\begin{aligned} \sigma(T, \xi) &= \beta \int_0^{|\xi|} N(E) dE \\ &+ \beta T \int_0^\infty \frac{dx}{1+e^x} [N(\xi+xT) \operatorname{sgn}(\xi+xT) - N(\xi-xT) \operatorname{sgn}(\xi-xT)] \end{aligned} \quad (8)$$

Conductivity (8) can also be rewritten as

$$\sigma(T, \xi) = \beta \int_0^\infty N(E) dE \left( \frac{1}{1+e^{(E-\xi)/T}} + \frac{1}{1+e^{(E+\xi)/T}} \right). \quad (9)$$

The last formula is very natural: the conductivity is proportional to the total number of charge carriers. For a single layer graphene  $N(E) = 2|E|/\pi(\hbar V)^2$  and for a bilayer (neglecting trigonal warping effects)  $N(E) = 2m/\pi\hbar^2$ . From (8) one has

$$\sigma(T, \xi) = \beta_s \left( \xi^2 + \frac{\pi^2}{3} T^2 \right) \quad (10)$$

for the single layer and

$$\sigma(T, \xi) = \beta_b \left( |\xi| + 2T \ln(1+e^{-|\xi|/T}) \right) \quad (11)$$

for the bilayer. For the single layer (6) takes the form

$$\int_0^\infty \left( \frac{1}{1+e^{x-y}} - \frac{1}{1+e^{x+y}} \right) x dx = \frac{\xi_0^2}{2T^2}, \quad (12)$$

where  $y = \xi(T)/T$ . The last equation should be solved numerically. For the bilayer (6) gives

$$n(T, \xi) = \frac{2m|\xi|}{\pi\hbar^2} = \frac{2m|\xi_0|}{\pi\hbar^2}. \quad (13)$$

Thus the chemical potential for the bilayer does not depend on temperature.

## Procedure for extracting of the peak broadening

Let us consider low concentrations  $n \rightarrow 0$ . From (6) we see that in this case  $\xi(T)/T \rightarrow 0$ .

Then the conductivity (9) can approximately be written as

$$\sigma(T, \xi) \approx \beta \left[ 2n_T(T) + \xi^2 \left( \frac{N(0)}{4T} + \frac{N'(0)}{2} + \int_0^\infty \frac{N''(E)}{1 + e^{E/T}} dE \right) \right], \quad (14)$$

where

$$n_T(T) = \int_0^\infty \frac{1}{1 + e^{E/T}} N(E) dE. \quad (15)$$

We note that the relation (14) holds for any density of states  $N(E)$ . This allows us to reconstruct of  $N(E)$  from conductivity measurements. For the single layer

$$n_T(T) = \frac{\pi T^2}{6(\hbar V)^2}. \quad (16)$$

For the bilayer

$$n_T(T) = \frac{2mT}{\pi \hbar^2} \ln 2. \quad (17)$$

For low concentrations (6) gives

$$n \approx \xi(T) \left( N(0) + 2 \int_0^\infty \frac{N'(E)}{1 + e^{E/T}} dE \right). \quad (18)$$

This gives

$$\xi(T) = \frac{\pi(\hbar V)^2}{4T \ln 2} n, \quad \xi(T) = \frac{\pi \hbar^2}{2m} n. \quad (19)$$

for single and bilayer respectively. Finally for the single layer

$$\sigma(T) \approx 2\beta n_T(T) \left[ 1 + \frac{\pi^2}{192(\ln 2)^2} \frac{n^2}{n_T^2(T)} \right], \quad (20)$$

and for the bilayer

$$\sigma(T) \approx 2\beta n_T(T) \left[ 1 + \frac{\ln 2}{8} \frac{n^2}{n_T^2(T)} \right]. \quad (21)$$



For the resistance we have  $R(T) = \sigma^{-1}(T)$ . One sees that for low concentrations resistance takes a form of a Lorentzian with a width

$$\Gamma(T) = \frac{8 \ln 2}{\sqrt{3}(\hbar V)^2} T^2 \quad (22)$$

for the single layer and

$$\Gamma(T) = \frac{8m\sqrt{2 \ln 2}}{\pi \hbar^2} T. \quad (23)$$

for the bilayer. At zero concentration the resistance decays as  $T^{-2}$  for a single layer and as  $T^{-1}$  for a bilayer.

At zero concentration we fit our experimental data by

$$\sigma(T) = \alpha + 2\beta n_T(T) \quad (24)$$

to find  $n_T(T)$ . Here  $\alpha$  takes into account the minimum conductivity at zero temperature.

The dependence  $n_T(T)$  is plotted in Fig. 2. For a bilayer we expect  $n_T(T)$  to obey (17), from what the effective mass can be extracted (see the main text).

## Energy spectrum in magnetic field

The Hamiltonian, describing the bi-layer graphene quasiparticle dynamics reads [2]:

$$H = \begin{pmatrix} 0 & \frac{\pi_-^2}{2m} + v_3 \pi_+ \\ \frac{\pi_+^2}{2m} + v_3 \pi_- & 0 \end{pmatrix}, \quad (25)$$

where  $\pi_{\pm} = \pi_x \pm i\pi_y$ ,  $\pi_j = -i\hbar \nabla_j + (e/c)A_j$ ,  $j = x, y$ ,  $e > 0$  is the absolute value of electron charge,  $c$  is the speed of light,  $A$  is the vector-potential. We introduce creation-annihilation Bose-operators  $b^+$ ,  $b$  as follows

$$\pi_+ = \sqrt{2e\hbar B/c} b^+, \quad \pi_- = \sqrt{2e\hbar B/c} b. \quad (26)$$

They satisfy the following commutation relation

$$[b, b^+] = 1. \quad (27)$$

The spectrum can be found from the equation  $H\psi = E\psi$ ,  $\psi = (\psi_1, \psi_2)$ . Excluding  $\psi_1$  from the last equation we obtain

$$[(b^+)^2 b^2 + \alpha((b^+)^3 + b^3) + \alpha^2 b b^+] \psi_2 = \varepsilon^2 \psi_2, \quad (28)$$

where  $\alpha = mv_3 \sqrt{2c/(e\hbar B)}$ ,  $\varepsilon = E/(\hbar\omega_B)$ ,  $\omega_B = eB/(mc)$ . In the basis of eigenfunctions  $|n\rangle$  of the operator  $b^+ b$  this equation takes the form  $M\eta = \varepsilon^2 \eta$ , where  $\eta = (\eta_0, \eta_1, \dots)$  and the matrix  $M$  is

$$M_{nn'} = [n(n-1) + \alpha^2(n+1)]\delta_{nn'} + \alpha \left[ \sqrt{(n+1)(n+2)(n+3)}\delta_{n+3,n'} + \sqrt{n(n-1)(n-2)}\delta_{n-3,n'} \right] \quad (29)$$

Eigenvalues  $\varepsilon^2$  of the sparse matrix  $M$  can be found using the Arnoldi method with a shift [3]. The shift is required due to the degeneracy of the zero eigenvalue.

For a weak magnetic field  $\alpha \ll 1$  and the distortion of Landau levels can be found analytically. Indeed we write  $(H_0 + H_1)\varphi = \varepsilon^2\varphi$ , where  $H_0 = (b^+)^2b^2 + \alpha^2bb^+$  and  $H_1 = \alpha((b^+)^3 + b^3)$ . The perturbation theory gives  $z_{n,0} = n(n-1) + \alpha^2(n+1)$ ,  $z_{n,1} = 0$ ,

$$z_{n,2} = \sum_{m \neq n} \frac{|\langle n | H_1 | m \rangle|^2}{z_{n,0} - z_{m,0}}, \quad (30)$$

Using (24) we find  $z_{n,2} = -\alpha^2(n+1)$ ,  $z_{n,0} + z_{n,2} = n(n-1)$ , i.e. eigenvalues are not perturbed up to  $1/B$ . The previous computation is valid for  $n > 2$ . For  $n \leq 2$

$$z_{n,0} + z_{n,2} = n(n-1) + \alpha^2n(1-n)/6. \quad (31)$$

Thus for  $n = 0,1$  we still get the absence of  $1/B$  correction. The only case when the correction arises is  $n = 2$ . Then  $z_{n,0} + z_{n,2} = 2 - \alpha^2/3$ ,  $E \approx \pm(\sqrt{2}\hbar\omega_B - \Delta E)$ . The estimation gives  $\Delta E = -\sqrt{2}mv_3^2/6 = -0.81 \text{ meV}$ . Thus

$$E_n \approx \pm(\hbar\omega_B\sqrt{n(n-1)} - \Delta E\delta_{n,2}). \quad (32)$$

Level spacings  $E_{n+1} - E_n$  corresponding to the plus sign for  $n = 1, \dots, 4$  are  $1.414\hbar\omega_B - \Delta E$ ,  $1.035\hbar\omega_B + \Delta E$ ,  $1.015\hbar\omega_B$ . Level spacings for higher levels rapidly tend to  $\hbar\omega_B$  since  $E_n \approx \hbar\omega_B$ . Thus, all level spacings are approximately equal to  $\hbar\omega_B$  apart from  $n = 0, 1, 2$ .

## Thermodynamic density of states and effective mass

Oscillations of magnetoresistance (Shubnikov-de Haas) and magnetization (Haas-van Alphen) caused by magnetic field are closely related to the “thermodynamic density of states” (TDOS)

$$D(\xi) = \int_{-\infty}^{\infty} N(E) \frac{\partial f(E - \xi)}{\partial \xi} dE, \quad (33)$$

where  $\xi$  is the chemical potential counted from the neutrality point. The Landau levels (LL) for bilayer graphene are  $\pm E_n$ ,  $E_n = \varepsilon_c \sqrt{n(n+1)}$ ,  $\varepsilon_c = \hbar\omega_B$ . Taking into account a broadening of LL, we have

$$n(E) = g_s g_v \frac{\Phi}{\Phi_0} \left\{ 2 \frac{\Gamma}{\pi} \frac{1}{E^2 + \Gamma_0^2} + \sum_{n=1}^{\infty} \left[ \frac{\Gamma}{\pi} \frac{1}{(E - E_n)^2 + \Gamma^2} + \frac{\Gamma}{\pi} \frac{1}{(E + E_n)^2 + \Gamma^2} \right] \right\}, \quad (34)$$

where  $g_s = g_v = 2$  are spin and valley degeneracies,  $\Phi, \Phi_0 = \hbar c / e$  are the total flux per sample and flux quantum, respectively and  $\Gamma$  is the broadening of LL. Factor 2,

corresponding to  $E_0 = 0$  reflects the degeneracy of the zero level. The Fourier component of  $\partial f(E - \xi) / \partial \xi$  can be computed using the residue theory:

$$\int_{-\infty}^{\infty} dE e^{iEt} \frac{\partial f(E - \xi)}{\partial \xi} = e^{i\xi t} \frac{\pi T t}{\sinh(\pi T t)}. \quad (35)$$

Then

$$D(\xi) = \int \int_{-\infty}^{\infty} \frac{dE dt}{2\pi} e^{i(\xi - E)t} \frac{\pi T t}{\sinh(\pi T t)} N(E). \quad (36)$$

We note that  $N(E)$  can be written in the form

$$N(E) = g_s g_v \frac{\Phi}{\Phi_0} \frac{\Gamma}{2\pi} \sum_{n=-\infty}^{\infty} \left[ \frac{1}{(E - E_n)^2 + \Gamma^2} + \frac{1}{(E + E_n)^2 + \Gamma^2} \right]. \quad (37)$$

From the Poisson summation rule

$$\sum_{n=-\infty}^{\infty} f(n) = \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} e^{2\pi i k x} f(x) dx \quad (38)$$

we obtain

$$N(E) = g_s g_v \frac{\Phi}{\Phi_0} \frac{\Gamma}{2\pi} \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} e^{2\pi i k x} f(x) dx, \quad (39)$$

where

$$f(n) = \frac{\Gamma}{\pi} \frac{E_n^2 + E^2 + \Gamma^2}{(E_n^2 - z_+^2)(E_n^2 - z_-^2)}, \quad (40)$$

and  $z_{\pm} = E \pm i\Gamma$ . The residue theory gives

$$\int_{-\infty}^{\infty} e^{2\pi i k x} f(x) dx = \frac{(-1)^k}{\varepsilon_c^2} \left[ \frac{z_+}{\alpha(z_+)} e^{i\pi|k|\alpha(z_+)} + \frac{z_-}{\alpha(z_-)} e^{i\pi|k|\alpha(z_-)} \right], \quad (41)$$

where  $\alpha(z) = \sqrt{4z^2 / \varepsilon_c^2 + 1}$ . The oscillating part of TDOS is

$$D_{osc}(\mu) = \frac{g_s g_v}{\pi \varepsilon_c^2} \frac{\Phi}{\Phi_0} \int \int_{-\infty}^{\infty} e^{i(\xi - E)t} \frac{\pi T t}{\sinh(\pi T t)} \sum_{k=1}^{\infty} (-1)^k [R_+(k, E) + R_-(k, E)] dt dE \quad (42)$$

$$, \\ R_{\pm}(k, E) = \frac{z_{\pm}}{\alpha(z_{\pm})} e^{\pm i\pi k \alpha(z_{\pm})}.$$

Assuming that  $S(t, E) = (\xi - E)t + \pi k \alpha(E) \gg 1$ ,  $\Gamma \ll \xi$  we can use the stationary phase method to compute  $D_{osc}(\xi)$  [4]. The result is

$$D_{osc}(\xi) = \frac{2g_s g_v}{\epsilon_c^2} \frac{\Phi}{\Phi_0} \frac{1}{\sqrt{1 + \epsilon_c^2 / 4\xi^2}} \sum_{k=1}^{\infty} (-1)^k Z(k) \cos\left(\frac{2\pi k \mu}{\epsilon_c} \sqrt{1 + \frac{\epsilon_c^2}{4\xi^2}}\right) e^{-\pi k \beta \Gamma}, \quad (43)$$

$$Z(k) = \frac{\pi k T \beta}{\sinh(\pi k T \beta)}, \quad \beta = \frac{2\pi m^* c}{\hbar e B}, \quad m^* = \frac{m}{\sqrt{1 + \epsilon_c^2 / 4\xi^2}}.$$

## Minimum conductivity of graphene and bilayer graphene

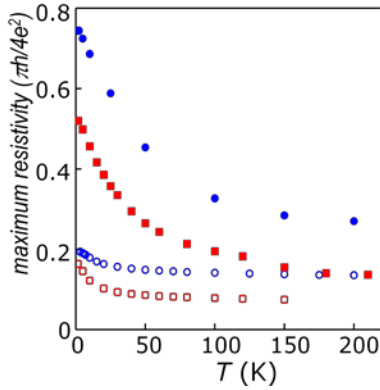


Fig. S1. Resistivity at the compensation point for two monolayer samples (solid symbols) and two bilayer devices (open symbols).

Our suspended samples, being nearly ballistic, demonstrate very different temperature dependence of the conductivity minimum in comparison to its non-suspended counterparts [7-8]. The minimum resistivity in low-mobility graphene on the substrate is usually higher or close to  $4e^2/h$  [7-8], which is a factor of  $\pi$  larger than that predicted by theory for ballistic transport [9-10]. Interestingly, the minimum conductivity for our suspended samples is approaching its theoretical value of  $4e^2/\pi h$  (Fig.

S1) which is in agreement with the previous results [11].

For ballistic bilayer graphene with parabolic dispersion relation the predicted minimum conductivity is  $8e^2/\pi h$  - twice that of monolayer graphene [12-13], whenever, if the effect of the trigonal warping is taken into account, the minimum conductivity is expected to be six times that of monolayer -  $24e^2/\pi h$  (4). Our experimental results demonstrate some intermediate value of about  $20e^2/\pi h$ , which might indicate the reduced number of pockets with linear dispersion around each of the K(K') points.

## Another Example of Bilayer Graphene Device

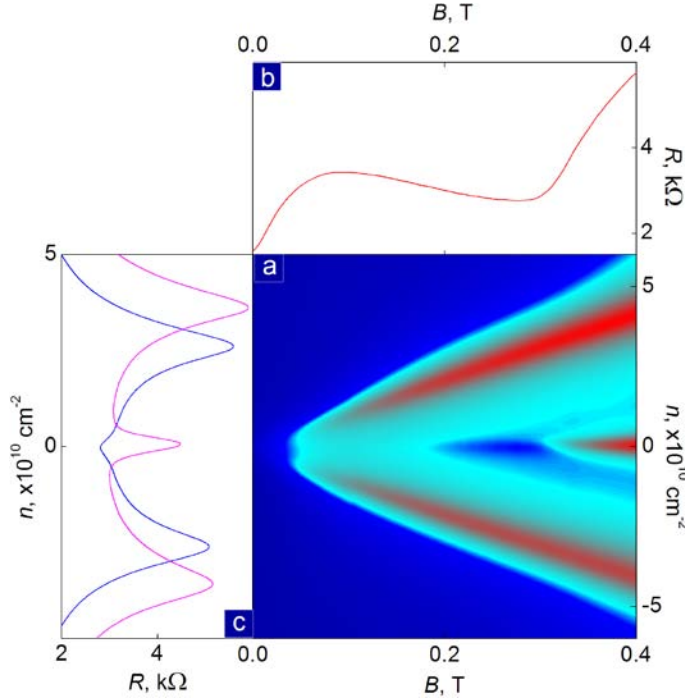


Fig. S2. Free-standing bilayer samples in magnetic field. a, Contour map of resistance of free-standing bilayer graphene device as a function of carrier concentration and magnetic field. The resistance spans from 2.1kΩ (blue) to 6kΩ (red). b, Resistance of suspended bilayer graphene at  $\nu=0$  as a function of magnetic field. Note negative differential magnetoresistance at approx  $0.1\text{T} < B < 0.3\text{T}$ . c, Resistance of suspended bilayer graphene as a function of gate voltage (recalculated to carrier concentration) for  $B=0.25\text{T}$  (blue); and  $B=0.35\text{T}$  (magenta). Note the sharp minimum on  $B=0.25\text{T}$  curve at  $-1 \times 10^9 \text{ cm}^{-2} < n < 1 \times 10^9 \text{ cm}^{-2}$ , which is replaced by a local maximum on  $B=0.35\text{T}$  curve.

To demonstrate how ubiquitous is the behavior we observed in our bilayer samples we present data from another bilayer sample with slightly lower mobility ( $\sim 700,000 \text{ cm}^2/\text{V}\cdot\text{s}$ ). All the characteristic features we observed for the sample presented in the main text can be seen here as well: (i) the resistance at filling factor zero is a nonmonotonous function of the magnetic field and diverges above 0.4T, which indicates the presence of a gap in the spectrum; (ii) the cyclotron gaps at filling factors  $\pm 4$  (maxima in the two-probe resistance) is significantly larger than the cyclotron gaps at other filling factors (so the oscillation at  $\nu = \pm 4$  survive for very low magnetic fields). The extracted gaps for different filling factors follow similar pattern

presented in the main text Fig. 4, just the plateau feature observed for  $\nu = \pm 4$  is at slightly lower energy ( $\sim 3 \text{ meV}$ ).

## Resonance-like dip at filling factor zero

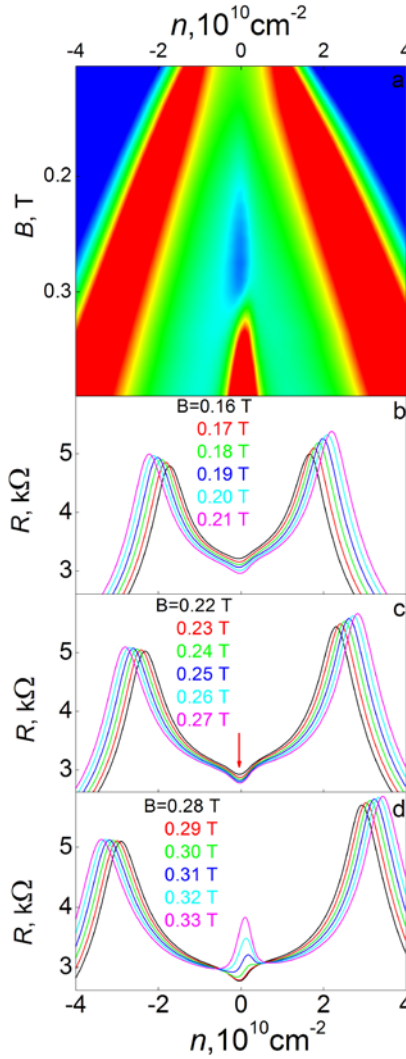


Fig. S3. A sharp local minimum (marked by the red arrow) appears for carrier concentrations  $3 \times 10^9 \text{ cm}^{-2} < n < 3 \times 10^9 \text{ cm}^{-2}$  and magnetic fields  $0.18 \text{ T} < B < 0.3 \text{ T}$ .

Magnetoresistance in our samples is a strongly non-monotonous function of magnetic field. For instance, for the samples presented on Fig. S2, magnetoresistance is positive for  $B < 0.1 \text{ T}$ , negative for  $0.1 \text{ T} < B < 0.3 \text{ T}$  and positive for  $B > 0.3 \text{ T}$ . Moreover, in the area of negative magnetoresistance a sharp, resonance-like dip is observed for carrier concentration  $-3 \times 10^9 \text{ cm}^{-2} < n < 3 \times 10^9 \text{ cm}^{-2}$  (Fig. S2, S3).

The negative magnetoresistance can be explained by the increase of the density of states in magnetic field at the compensation point due to formation of zero Landau level (at  $B=0$  the density of states at the Dirac point is zero for the linear spectrum). Thus, the conductivity should rise from the quantum limited value (for the unreconstructed spectrum it was estimated [5-6] as  $24e^2/\pi h$ ) to that determined by the density of states at the zero Landau level.

At the moment we haven't got an explanation for the appearance of the sharp dip in the resistance, but would like to point out that the range of carrier concentration occupied by the dip corresponds roughly to the number of states for within the conical part of the bilayer spectrum.

## References

- [1] A. Isihara, *Statistical Physics* (Acad. Press, New York, 1971).
- [2] E. McCann, and V. I. Fal'ko, Phys. Rev. Lett. **96**, 086805 (2006).
- [3] W. E. Arnoldi, Quarterly of Applied Mathematics **9**, 17 (1951).
- [4] M. V. Fedoryuk, *Saddle-point method* (Nauka, Moscow, 1977).
- [5] J. Cserti, A. Csordas, and G. David, Phys. Rev. Lett. **99**, 066802 (2007).
- [6] M. Trushin *et al.*, Phys. Rev. B **82**, 155308 (2010).
- [7] K. S. Novoselov *et al.*, Nature **438**, 197 (2005).
- [8] A. K. Geim, and K. S. Novoselov, Nature Mater. **6**, 183 (2007).
- [9] M. I. Katsnelson, European Physical Journal B **51**, 157 (2006).
- [10] J. Tworzydło *et al.*, Phys. Rev. Lett. **96**, 246802 (2006).
- [11] F. Miao *et al.*, Science **317**, 1530 (2007).
- [12] J. Cserti, Phys. Rev. B **75**, 033405 (2007).
- [13] I. Snyman, and C. W. J. Beenakker, Phys. Rev. B **75**, 045322 (2007).