In Edwin Abbott’s 1884 novella *Flatland: A Romance of Many Dimensions*, the narrator (a square, of course) leads the reader through some of the special features of two dimensions. As the geometric characters in the story come to learn, dimension matters—a lesson that has not been lost on 20th- and 21st-century physicists. Although we can readily imagine a world with fewer or more dimensions and describe it mathematically, it seems at first glance that for natural phenomena we are stuck with three spatial dimensions and one time dimension. Not so! For many years now, for example, physicists have studied electronic properties of the two-dimensional systems that occur in layered semiconductors, and not without reward, since the operation of every computer chip today relies on properties of the electronic flatland at the interface between silicon and its oxide.

Any real flatland must have a finite thickness. In the case of semiconductors, the thickness typically extends from 10 to 100 atomic layers and the system can accurately be described as 2D only because of quantum size effects that make the degrees of freedom for electron motion in the short direction irrelevant. A few years ago a research group at the University of Manchester led by one of us (Geim) succeeded in isolating and studying the ultimate flatland—graphene, a one-atom thick sheet of carbon atoms arranged laterally in a honeycomb lattice.¹ (For a primer on graphene’s crystal and electronic structures, see box 1.) This flatland is not only the thinnest material in our universe, but also so charming in its properties that it had already been the object of theoretical study for more than half a century² before it at last became available for experimental inspection.

**Graphene spotting**

Fundamental forces place seemingly insurmountable barriers in the way of creating a true flatland. In the case of graphene,
chemical synthesis has so far yielded only tiny platelets some 10 benzene rings across, containing up to 222 carbon atoms. Conventional crystal growth techniques are also of little help, since thermal fluctuations at growth temperatures tend to twist otherwise flat nanometer-scale graphene crystallites into the third dimension. The nascent 2D crystallites try to minimize their surface energy and inevitably morph into one of the rich variety of stable 3D structures that occur in soot.

But there is a way around the problem. Interactions with 3D structures stabilize 2D crystals during growth. So one can make 2D crystals sandwiched between or placed on top of the atomic planes of a bulk crystal. In that respect, graphene already exists within graphite, which can be viewed as a stack of graphene layers stuck together by van der Waals–like attraction. One can then hope to fool Nature and extract single-atom-thick crystallites at a low enough temperature that they remain in the quenched state prescribed by the original higher-temperature 3D growth. Unfortunately, no one has invented tweezers small enough to pull individual atomic planes from bulk crystals.

It turns out that graphene sheets, like fullerenes and carbon nanotubes, have always been around. Right before our eyes, in fact. In the simple trace of a pencil is debris composed mostly of readily visible, thick graphite flakes rubbed from the bulk crystal. But thinner, very nearly transparent crystallites—some a single layer thick—are also present. The hard part is not making graphene but finding it in the haystack of thicker flakes.

Typically, a few micron-sized graphene crystallites exist in a graphite debris field covering an area of 1 cm². Scanning electron microscopy is of little help in the search because it cannot distinguish monolayers from nanometer-thick flakes. Atomic-force and scanning-tunneling microscopes provide the required atomic resolution but can usually detect the step between a substrate and a monolayer only when the substrate is atomically smooth. Even more forbidding is the need to scan the entire area of a pencil trace with atomic resolution.

The serendipitous choice in the Manchester lab for finding graphene was to use not paper, or any other writing surface, but an oxidized Si wafer—the same material widely used by the semiconductor industry. The oxide surface reflects a rainbow of colors, and the interference pattern produced by layers of graphene on the oxide provides a faint but visible contrast, much like the fringes in an oily puddle (see figure 1). Fortunately, the human eye and brain are a team powerful enough to distinguish even that weak contrast in rapid optical microscope inspections of graphite debris. With a little experience, finding those few graphene crystallites takes only a couple of hours.

No one usually uses pencils to make graphene. Instead, to make graphene crystals suitable for experiments, bulk graphite is gently pushed along a Si wafer, a “drawing” technique that several laboratories have now refined to the level of art. Graphene crystallites as large as 100 μm across can be formed this way and also obtained commercially (see http://www.grapheneindustries.com).
The valence and conduction bands of graphene are generally described by a $2 \times 2$ Hamiltonian matrix $H_{X \times Y}(p)$, where $X'$ and $X$ label sublattices A and B, and $p$ represents the crystal momentum in the lattice’s Brillouin zone. It is convenient—and common in the graphene literature—to regard the sublattice degree of freedom as a pseudospin, with the A sublattice being the $|\uparrow\rangle$ pseudospin state and the B sublattice being the $|\downarrow\rangle$ pseudospin state. The hopping of electrons between sublattices produces an effective magnetic field that is proportional in magnitude and direction to momentum measured from the Brillouin-zone corners.

By writing $H_{X \times Y}(p) = -h(p) \cdot \tau_{X \times Y}$, where $\tau$ is the Pauli spin matrix vector, the Hamiltonian can be viewed in terms of a momentum-dependent magnetic field $h$ that acts on the pseudospin degree of freedom. For each $p$, the pseudospin in the lower-energy valence-band state points along $h$, and the pseudospin in the higher-energy conduction-band state points in the opposite direction. The splitting between conduction and valence bands vanishes when the pseudospin field $h(p)$ is zero. Because $\tau_x$ is diagonal and $\tau_y$ and $\tau_z$ are purely off-diagonal, $h_y(p)$ accounts for the difference between $H_{Aa}(p)$ and $H_{Ba}(p)$, while $h_z(p)$ and $h_x(p)$ account for the hopping of electrons between sublattices.

In graphene, $H_{Aa}(p) = H_{Ba}(p)$ because the A and B sublattices are identical, so $h_z(p)$ vanishes by symmetry everywhere in the Brillouin zone. The A lattice sites that contribute to $H_{Aa}(p)$ occur in symmetry-equivalent triplets that are related by 120° rotations about a B site. It turns out that at the two nonequivalent Brillouin-zone corner points $K$ and $K'$, the contributions to $H_{Aa}(p)$ from the members of each triplet are mutually out of phase by $2\pi/3$ and sum to zero. At $K$ and $K'$, then, $h_x$ and $h_y$ also vanish. The conduction and valence bands therefore touch at those high-symmetry points, which makes graphene a gapless semiconductor.

The relative changes in electron density that occur from doping graphene with charged impurities or applying an external electric field are fairly small. As a result, graphene’s electronic properties depend mainly on the energy bands near $K$ and $K'$. Writing the momentum as a sum of wave vectors $p = K + k$ near the $K$ valley, we can therefore assume that $k$ is small and expand to leading order in this vector parameter. Using the symmetry considerations outlined above, it follows that $H_{Aa}(p) \approx v_k \exp(i[\phi_k - \phi_0])$, where $\phi_k$ specifies the planar orientation of $k$, the phase $\phi_0$ is $k$-independent, and the velocity $v_k$ depends on microscopic details. The $\pi$-orbital band Hamiltonian for each spin and valley can therefore be written in the form $H = -v_k \hbar \mathbf{k} \cdot \mathbf{\tau}$. That is, the effective magnetic field, which acts on the pseudospin, points in the same direction as the momentum $\mathbf{k}$. And the band eigenstates have definite projection of pseudospin chirality.

Following the standard tricks of condensed-matter physics, in the presence of a slowly varying external potential or magnetic field the momentum $\mathbf{k}$ becomes a quantum-mechanical momentum operator. When both valleys are treated at once, the band spinors have four components, much like the spinors that describe electrons and positrons in the Dirac equation. As a result, the band Hamiltonian of graphene turns out to be identical to the zero-mass limit of the Dirac equation.

High-quality single crystals some thousands of square microns in size are sufficient for most fundamental physics studies, proof-of-concept devices, and, possibly, even complex electronic circuits; for a recent technical review, see reference 3. However, for industrial-scale applications, the drawing technique is almost certainly not viable. Significant efforts are under way to grow graphene epitaxially by thermal decomposition of silicon carbide or by vapor deposition of hydrocarbons on catalytic metallic surfaces that could later be etched away to leave graphene on an insulating substrate. Graphene can also be obtained in powder form, by exfoliation of “graphene oxide” from graphite oxide followed by reduction to graphene. Although future applications of graphene will depend on progress with its epitaxial growth, the physics of this new 2D electron system is already fully accessible.

**Slow relativity**

As explained in box 1, graphene is a gapless semiconductor in which the valence- and conduction-band energies are linear functions of momentum. This property implies that the speed of electrons in graphene is a constant, independent of momentum, much like the speed of photons is a constant $c$. In fact, the quantum mechanics of graphene’s electrons is identical to the quantum mechanics of relativistic particles with a vanishingly small mass, and its free-particle states are therefore chiral. For spin-1/2 quantum particles chirality is akin to helicity, the quantized projection of a particle’s spin along its direction of motion (see box 2). Graphene’s relativistic behavior arises not from required consistency with special relativity—or more specifically with Lorentz invariance—but simply from the symmetry of the honeycomb lattice.

Because of the similarity of its Hamiltonian with that of massless, relativistic particles, graphene has many properties that can be anticipated just by browsing a textbook on relativistic quantum mechanics. The Dirac equation tells us, for example, that by transforming themselves into virtual antiparticles, relativistic particles can penetrate a potential barrier of any height and width without creating a reflected component. This effect, called the Klein paradox, is one of the most counterintuitive phenomena in QED. Predicted decades ago, it has never been observed in particle physics experiments. But in graphene the reflectionless tunneling of Dirac fermions through potential barriers and the unhindered conversion between electron- and hole-like charge carriers occur routinely and serve to enhance graphene’s conductivity.

The velocity $v$ of graphene’s electrons is about $10^6$ m/s, large but still some 300 times slower than $c$. Because the electrons are sluggish compared to the speedy photons they exchange when interacting, the physics of electron–electron interactions in graphene is different from that of photon-mediated interactions between fermions in QED. In graphene the interactions among electrons are expected to be extremely strong because their mutual screening is weaker than in metals and because graphene’s dimensionless coupling constant $\alpha_{\mathbf{g}} = e^2 / \hbar v \approx 1$ is much larger than the dimensionless coupling constant of QED—the fine structure constant $\alpha = e^2 / \hbar c \approx 1/137$. The large difference between $c$ and $v$ also implies that the model that describes interacting electrons in a graphene sheet is, unlike the 2D version of QED, not Lorentz invariant. In fact, interactions between electrons in
graphene lead to a strong enhancement of the particle velocity that would be ruled out by symmetry in a Lorentz-invariant theory. As indicated in figure 2, the role of interactions is also different from their role in an ordinary nonrelativistic 2D electron gas. All this makes graphene an intriguing new type of electronic system whose independent particles move relativistically, but interact nonrelativistically. The properties of this chiral 2D electron gas are just beginning to be understood.

Unstoppable electrons

To appreciate how unusual the Dirac fermion system is, consider its conductivity at the neutrality point, where the valence and conduction bands meet. Since the density of electrons or holes vanishes as the Fermi energy approaches zero, the natural expectation is that the conductivity should also vanish in this limit. Experiment flouts that expectation. Graphene has never exhibited a resistivity larger than several kilohms, and attempts to make it less conductive have so far failed. Eduardo Fradkin predicted this counterintuitive behavior in 1986. His theory argues that as the Fermi energy approaches zero, even the very last electron or hole provides a minimum conductivity of about $e^2/h$, irrespective of whether a graphene sheet measures one square micron or one square mile.

Part of what explains the nonzero conductivity is the difficulty of localizing massless Dirac fermions, which can slip through a rough potential landscape that would trap a nonrelativistic electron. Indeed, massless Dirac fermions do not make bound states in a shallow 2D potential, unlike normal Schrödinger fermions that do, no matter how weak the potential. (This is the Klein paradox in action in graphene.) Because graphene’s electrons are so hard to localize, it is tempting to appeal to an old argument of Nevill Mott’s: Like light, an electron in a metal does not notice any roughness on a scale shorter than its wavelength $\lambda$ and therefore can never have a mean free path $l$ smaller than $\lambda$. Setting $l \approx \lambda$ then, standard Boltzmann transport theory implies that $\sigma = (e^2/h)(l/\lambda) \approx e^2/h$.

The simple argument assumes that Dirac fermions remain delocalized, which contradicts the notion that electron localization is inevitable in two dimensions. Most theorists expect that the resistivity will eventually start increasing at some ill-defined ultralow temperature, but that has not yet been observed. The apparent suppression of localization remains one of graphene’s mysteries. Strong interactions among electrons near the neutrality point might play a role. But while we search for deeper understanding, Mott’s argument provides some comfort for our intuition.

Quantum Hall variations

In the presence of a perpendicular magnetic field, electrons confined to a plane experience a Lorentz force that constrains their motion to closed cyclotron orbits. According to quantum theory, the circumferences of those orbits must contain an integer number of de Broglie wavelengths. That requirement gives rise to a discrete spectrum of allowed kinetic energies known as Landau levels, each of which is macroscopically degenerate because the orbits can be centered anywhere in the 2D plane. Consequently, all electronic properties of clean, 2D systems become quantized.

Magnetotransport measurements of 2D systems dramatically bear out this quantization. Two years ago two groups—Philip Kim and colleagues at Columbia University and Kostya Novoselov, one of us (Geim), and colleagues at the University of Manchester—independently observed the quantum Hall effect in graphene. The achievement was an important milestone because it demonstrated that the single-layer sheets have the characteristic and unusual properties of nearly perfect honeycomb crystals (see figure 3).

The quantum Hall effect allows us to make inferences about the electronic spectrum based on transport data as a function of charge-carrier density $n$. Minima in longitudinal resistivity $\rho_n$ identify densities at which Landau levels are full. The fact that the sequence of $\rho_n$ maxima in figure 3 is centered on $n = 0$ implies that a Landau level lies right at the neutrality point, $E = 0$. Note that in zero magnetic field no states exist at this energy. Furthermore, we know from our experience with the quantum Hall effect that the plateau values of the transverse conductivity $\sigma_{xy}$ measure the densities at which Landau levels are full. The plateaus at $\sigma_{xy} = \pm 2e^2/h$ in figure 3a indicate that the $E = 0$ level takes only half the usual number of states from the conduction band and draws the other half from the valence band. This unusual property
of the zero Landau level is responsible for the anomalous sequence of $\sigma_{xy}$ plateaus in graphene. The sequence of Landau levels is shifted by $1/2$, compared with the standard quantum Hall effect, so that $\sigma_{xy} = \pm 4e^2/h(N + \frac{1}{2})$, where $N$ is the Landau level index and the factor 4 accounts for graphene's spin and band degeneracy.

The quantum Hall effect in bilayer graphene is even more interesting. The magnetic field draws two Landau levels to $E = 0$, one from the conduction band and one from the valence band. The quantized plateaus now appear at integer states, positrons become valence-band holes, and the mass plays the role of spin. Electrons become conduction-band honeycomb-lattice pseudospin degree of freedom (see box 2).

Finally, another intriguing aspect of graphene's quantum Hall effect is the observation that in sufficiently strong magnetic fields additional gaps open up in graphene's spectrum at partial Landau-level occupancies. These gaps are a consequence of broken symmetries. The most interesting example is the additional plateau that appears in an electric field, forces the majority spins into the lowest-energy conduction-band gap and the minority spins into the highest-energy valence-band gap. True fractional quantum Hall effects have not yet been observed in graphene, but they are expected as sample quality improves. With that improvement, additional many-body physics surprises are sure to come.

A soft solid

Graphene is the first example of truly atomic 2D crystalline matter. Because graphene can be prepared as a suspended sheet, the lush physics of soft membranes may now be explored in new ways. It wouldn't be a surprise to discover that the structural properties of this flatland are as enchanting as its electronic ones.

The mechanical resonance frequencies of any suspended membrane are, like those of a drum head, proportional to the square root of applied tension. When very loosely suspended,
biological membranes and other thin sheets become floppy and tend toward buckling and crumpling (see the article by Michael Marder, Robert Deegan, and Eran Sharon, PHYSICS TODAY, February 2007, page 33). Floppiness is a consequence of flatness because slow variations in a membrane’s local orientation correspond to sheet rotations that cost little energy. The sheet’s response to thermal fluctuations and external stress is extremely rich. For example, researchers expect atomically thin membranes to have a flat phase in which anharmonic interactions with in-plane vibrations drive the sheet’s bending modulus to high values at long length scales and suppress macroscopic crumpling. This is an example of order-from-disorder phenomena, common in frustrated magnets and other complex systems. Recent studies of very loosely suspended graphene sheets show that they are indeed flat, with ripples only a few angstroms in height and several nanometers in length, much smaller than the typical sample size. Although these experimental results are not yet fully understood, it is tempting to view them as evidence for the glass-like state that researchers expect when the flat phase is subjected to external stress.

The crumpling physics of a graphene sheet is closely tied to its electronic properties. The symmetries of the honeycomb lattice are lifted by crumpling: The A and B sublattices become locally distinguishable, and the hopping amplitudes of an electron to an A site from its three B sublattice neighbors no longer cancel at the Brillouin-zone corner. Importantly, both the energy and the wave vector position of the neutrality point fluctuate. The effect is equivalent to graphene’s Dirac fermions moving in random electric and vector potentials. Crumpling therefore can be important for electron transport and may explain a number of unusual experimental results—for example, the observed suppression of weak localization in doped graphene.

**Flatland to the rescue**

What kind of mind-boggling technology might emerge from graphene? Before we proffer an answer, imagine you are on a boat trip watching a school of dolphins. Everyone is mesmerized by the magnificent animals until someone spoils the moment by voicing the unromantic question, “But can we eat them?” One-atom-thick materials have only recently been spotted in our universe and most researchers are happy, for the moment, to expand our understanding of this new and captivating type of matter. What can be said with confidence about applications is only that graphene holds as much promise as one could possibly hope for.

That optimism is based partly on recent progress in epitaxial growth of graphene and in its bulk powder production, and partly on the many possible ways of using graphene. More than a decade of research on carbon nanotubes—essentially rolled-up graphene—has already led to many applications. Based on the similarities between the two carbon allotropes, a multitude of applications come to mind: exploiting graphene’s high surface-to-volume ratio and high conductivity in composite materials and in electrical batteries, its atomic thinness in field emitters and in transparent membranes for electron microscopy, its robustness and light weight in micromechanical resonators, and its tunability by an external electric field in superconducting and spin-valve transistors and in ultra-sensitive chemical detectors. Those are just a few of the possibilities.

Graphene’s largest impact is likely to be in electronics. Its electrons move ballistically over submicron distances, even under ambient conditions. With their high speed, the electrons take less than 0.1 ps to cover the typical distance between source and drain electrodes in a transistor. It should not be long before graphene-based ultrahigh-frequency transistors are demonstrated and—pending the development of graphene wafers—find their way into electronics.

Even more tantalizing is the hope graphene offers as an eventual replacement for silicon in ever tinier integrated circuits. Even partisans of Si-based technology expect that Si’s fundamental limits will be reached in 15 to 20 years, when individual circuitry components shrink to 10 nm. Graphene can be scaled to extremely fine scales, possibly down to a single benzene ring, because unlike other materials, such as Si or metals used in the electronics industry, graphene remains stable and conductive at the molecular scale.

Currently many groups are studying graphene nanoribbons, ultranarrow strips of the material in which a semiconducting gap can be opened due to quantum confinement of electrons. In principle, that effect should allow graphene-based transistors to operate in the same way as Si-based ones. The strategy for graphene-ribbon electronics can be likened to what’s currently attempted with nanotubes: controlling their size, type, and placement in an electronic circuit. But rather than using a set of nanotubes as ingredients for a circuit, engineers could carve the entire circuit out of graphene—the bulk electrodes, quantum barriers, central molecular switches, and quantum dots. At the single-nanometer scale, a few benzene rings cut out of a graphene sheet can leave behind a macromolecule attached to graphene-based electrical contacts, as suggested in figure 4.

The central graphene island in this example can be considered as a quantum dot, separated from macroscopic contacts by either resistive or tunneling barriers. Making graphene circuitry operational at room temperature on such quantum principles will require dimensions smaller than 10 nm. In theory, the smaller the graphene elements, the better they
should serve electronics applications. In practice, that’s hard to achieve. But for the first time, a material’s properties do not rule out miniaturization at such fine scales.

It is premature to predict whether graphene will indeed emerge as the replacement to today’s Si circuit. But certainly no other material has spoiled researchers and engineers with as many appealing angles to pursue.

References